

Lecture 2  
2022/2023

# Microwave Devices and Circuits for Radiocommunications

# 2022/2023

- 2C/1L, **MDCR**
- Attendance at minimum 7 sessions (course or laboratory)
- Lectures- **associate professor Radu Damian**
  - Tuesday 12-14, ~~Online~~, P8
  - E – 50% final grade
  - problems + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 21-28.02.2023 (t<sub>2</sub> and t<sub>3</sub> not announced, lecture)
    - 3att.=+0.5p
  - all materials/equipments authorized

# 2022/2023

- Laboratory – **associate professor Radu Damian**
  - Tuesday 08-12, II.13 / (08:10)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 21.02.2022)
    - personal homework

# Materials

■ <http://rf-opto.etti.tuiasi.ro>

The screenshot shows a web browser displaying the website [http://rf-opto.etti.tuiasi.ro/microwave\\_cd.php?ch\\_lang=0](http://rf-opto.etti.tuiasi.ro/microwave_cd.php?ch_lang=0). The page title is "Microwave Devices and Circuits for Radiocommunications (English)". The main content area includes sections for Course (MDCR 2017-2018), Activities, Evaluation, Grades, Attendance, Lists, and Materials. The right side features the RF-OPTO logo, a globe graphic, and language links (English, Romana). A red circle highlights the "English" link in the language bar.

Laboratorul de Microunde și Optică

Main Courses Master Staff Research Students Admin

Microwave CD Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

## Microwave Devices and Circuits for Radiocommunications (English)

**Course:** MDCR (2017-2018)

**Course Coordinator:** Assoc.P. Dr. Radu-Florin Damian  
**Code:** EDOS412T  
**Discipline Type:** DOS; Alternative, Specialty  
**Credits:** 4  
**Enrollment Year:** 4, Sem. 7

**Activities**

**Evaluation**

Type: Examen

A: 50%, (Test/Colloquium)  
B: 25%, (Seminary/Laboratory/Project Activity)  
D: 25%, (Homework/Specialty papers)

**Grades**

[Aggregate Results](#)

**Attendance**

[Course](#)  
[Laboratory](#)

**Lists**

[Bonus-uri acumulate \(final\)](#)  
[Studentii care nu pot intra in examen](#)

**Materials**

**Course Slides**

MDCR Lecture\_1 (pdf, 5.43 MB, en, )  
MDCR Lecture\_2 (pdf, 3.67 MB, en, )  
MDCR Lecture\_3 (pdf, 4.76 MB, en, )  
MDCR Lecture\_4 (pdf, 5.58 MB, en, )

RF-OPTO

ETTI

English | Romana |

Main Courses Master Staff Research

Grades Student List Exams Photos

## Online Exams

In order to participate at online exams you must get ready following:

# Materials

- RF-OPTO
  - <http://rf-opto.eti.tuiasi.ro>
- **David Pozar, “Microwave Engineering”,**  
Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by **email**/online exam
  - used at lectures/laboratory

# Photos



**Date:**

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

[Trimite email acestui student](#) | [Adauga acest student la lista \(0\)](#)

<b>Detalii curente</b>	<b>Observatii</b>
Finantare	Buget
Bursa	Fara Bursa



**Date:**

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5244

[Trimite email acestui student](#) | [Adauga acest student la lista \(0\)](#)

<b>Detalii curente</b>	<b>Observatii</b>
Finantare	Buget
Bursa	Bursa de Studii



**Date:**

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

[Acceseaza ca acest student](#)

**Note obtinute**

Disciplina	Tip	Data	Descriere	Nota	Puncte	Obs.
TW	Tehnologii Web					
	N	17/01/2014	Nota finala	10	-	
	A	17/01/2014	Cale vid Tehnologii Web 2013/2014	10	7.55	
	B	17/01/2014	Laborator Tehnologii Web 2013/2014	9	-	
	D	17/01/2014	Tema Tehnologii Web 2013/2014	9	-	

# Profile photo

- Profile photo – online “exam”

**Examene online: 2020/2021**

**Disciplina: MDC (Microwave Devices and Circuits (Engleza))**

**Pas 3**

Nr.	Titlu	Start	Stop	Text
1	Profile photos	03/03/2021; 10:00	08/04/2021; 08:00	Online "exam" created f ..
2	Mini Test 1 (lecture 2)	03/03/2021; 15:35	03/03/2021; 15:50	The current test consis ..



**Grupa** 5304 (2015/2016)

**Specializarea** Tehnologii si sisteme de telecomunicatii

**Marca** 5184

# Access

- Not customized

A screenshot of a student profile page. On the left is a thumbnail photo of a student. Below it is a link "Acceseaza ca acest student". To the right is a section titled "Date:" containing the following information:

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

Below this is a section titled "Note obtinute" with a table:

Disciplina	Tip	Data	Descriere	Nota	Puncte	Obs.
TW	Tehnologii Web					
	N	17/01/2014	Nota finala	10	-	
	A	17/01/2014	Colocviu Tehnologii Web 2013/2014	10	7.55	
	B	17/01/2014	Laborator Tehnologii Web 2013/2014	9	-	
	D	17/01/2014	Tema Tehnologii Web 2013/2014	9	-	

A screenshot of a contact form. It includes fields for "Nume" (Name) with a redacted value, "Email" (Email), and "Cod de verificare" (Verification code) with a redacted value. At the bottom is a large blue button containing the text "344bd9f". A red arrow points from the "Email" field on the left to the verification code button on the right.

Nume  
MOOROUN

Email

Cod de verificare

344bd9f

Trimite

# Online

- access to **online exams** requires the **password** received by email

English | Romana |

Main Courses Master Staff Research **Student List**

Grades Student List Exams Photos

## POPESCU GOPO ION

Fotografia nu există

Date:

Grupa	5700 (2019/2020)
Specializarea	Inginerie electronica si telecomunicatii
Marca	7000000

[Access the site as this student](#) | [Request access to software](#)

**Grades**

Inca nu a fost notat.

Main Courses Master Staff Research

Grades **Student List** Exams Photos

### Login

Use the last name and email stored in the database

Name  
POPESCU GOPO

Email/Password

Write the code below

828f26b

Send

# Online

- access email/password

Main Courses Master Staff Research

Grades Student List Exams Photos

## POPESCU GOPO ION

**Fotografia nu există**

Date:

Grupa	5700 (2019/2020)
Specializarea	Inginerie electronica si telec
Marca	7000000

You access the site as **this student!**

Main Courses Master Staff Research

Grades Student List Exams Photos

## POPESCU GOPO ION

**Fotografia nu există**

Date:

Grupa	5700 (2019/2020)
Specializarea	Inginerie electronica si telec
Marca	7000000

You access the site as **this student (including exams)!**

# Password

## ■ received by email

Important message from RF-OPTO Inbox x

Radu-Florin Damian  
to me, POPESCU ▾

Romanian ▾ English ▾ Translate message

 Laboratorul de Microunde si Optoelectronica  
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei  
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul rf-opto este  
Parola: [REDACTED]

Identificati-vă pe [server](#), cu parola, cat mai rapid, pentru confirmare.

**Memorati** acest mesaj intr-un loc sigur, pentru utilizare ulterioara

---

Attention: POPESCU GOPO ION

The password to access the exams on the rf-opto server is  
Password: [REDACTED]

Login to the [server](#), with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Reply Reply all Forward

Subject: Important message from RF-OPTO Correspondents

From Me <[rdamian@etti.tuiasi.ro](mailto:rdamian@etti.tuiasi.ro)> ★  
Subject: **Important message from RF-OPTO** ★

To [REDACTED] ★  
Cc Me <[rdamian@etti.tuiasi.ro](mailto:rdamian@etti.tuiasi.ro)> ★

 Laboratorul de Microunde si Optoelectronica  
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei  
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul **rf-opto** este  
Parola: [REDACTED]

Identificati-vă pe [server](#), cu parola, cat mai rapid, pentru confirmare.

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Attention: POPESCU GOPO ION

The password to access the exams on the **rf-opto** server is  
Password: [REDACTED]

Login to the [server](#), with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

# Online exam manual

- The online exam app used for:
  - ~~lectures (attendance)~~
  - laboratory
  - project
  - ~~examinations~~

## Materials

### Other data

[Manual examen on-line \(pdf, 2.65 MB, ro, !\[\]\(65669ef2a9341eca7c5ba6092e766555\_img.jpg\)](#)

[Simulare Examen \(video\) \(mp4, 65.12 MB, ro, !\[\]\(7f8d804c6d199749d3dd53592a5ca12b\_img.jpg\)](#)

# Examen online

- always against a **timetable**
  - long period (lecture attendance/laboratory results)
  - ~~short period (tests: 15min, exam: 2h)~~

Announcement 23:59 (10/05/2020)	Support material 00:05 (11/05/2020)	Exam Topics 00:07 (11/05/2020)	Results 00:10 (11/05/2020)	End 00:20 (15/05/2020)	Confirmation 00:20 (16/05/2020)	Next timeframe in: <b>05 m 43 s</b> <a href="#">Refresh now</a>
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**Announcement**

This is a "fake" exam, introduced to familiarize you with the server interface and to perform the necessary actions during an exam: thesis scan, selfie, use email for co...

**Server Time**

All exams are based on the server's time zone (it may be different from local time). For reference time on the server is now:

**10/05/2020 23:59:16**

# Online results submission

- many numerical values/files

Schema finala	Rezultate - castig	Rezultate - zgromot	Fisier justificare calcul (factor andrei)	Fisier zap (optional)	T1, fisier parmetri S	T2, fisier parmetri S	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Ze1	Zo1	Ze2	Zo2	Ze3	Zo3	Ze4	Zo4	Ze5	Zo5	Ze6
86 - 5428 - 259 ...	86 - 5428 - 260 ...	86 - 5428 - 261 ...	86 - 5428 - 316 ...	-	86 - 5428 - 314 ...	86 - 5428 - 315 ...	148.33	155.88	202.12	164.35	180.91	30.29	185.19	79.9	37	68.89	45.14	61.83	45.05	57.97	46.02	61.85	45.05	68.8
86 - 5622 - 259 ...	86 - 5622 - 260 ...	86 - 5622 - 261 ...	86 - 5622 - 316 ...	86 - 5622 - 262 ...	86 - 5622 - 314 ...	86 - 5622 - 315 ...	26.97	153.5	34.64	35.79	55.56	26.212	10.693	0	0	0	0	0	0	0	0	0	0	0
86 - 5488 - 259 ...	86 - 5488 - 260 ...	86 - 5488 - 261 ...	86 - 5488 - 316 ...	86 - 5488 - 262 ...	86 - 5488 - 314 ...	86 - 5488 - 315 ...	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
86 - 5391 - 259 ...	86 - 5391 - 260 ...	86 - 5391 - 261 ...	86 - 5391 - 316 ...	-	-	-	50	50	50	50	50	50	50	70.14	40.39	61.85	44.59	55.7	45.2	54.89	45.38	58.65	45.8	70.0
86 - 5664 - 259 ...	86 - 5664 - 260 ...	86 - 5664 - 261 ...	86 - 5664 - 316 ...	-	86 - 5664 - 314 ...	86 - 5664 - 315 ...	168.02	150.5	178.28	133.75	92.12	121.67	144.48	94.36	36.19	70.77	42.56	65.69	42.05	55.17	42.29	65.59	42.05	70.7
86 - 5665 - 259 ...	86 - 5665 - 260 ...	86 - 5665 - 261 ...	86 - 5665 - 316 ...	-	86 - 5665 - 314 ...	86 - 5665 - 315 ...	162.2	80.8	209.2	140.85	135.1	183.7	167.6	94.58	36.15	78.16	39.77	65.57	45.05	65.57	45.05	78.16	39.77	94.5
86 - 5433 - 259 ...	86 - 5433 - 260 ...	86 - 5433 - 261 ...	86 - 5433 - 316 ...	-	86 - 5433 - 314 ...	86 - 5433 - 315 ...	165.138	106.228	226.157	130.134	72.71	180.177	164.616	101.36	36.11	77.22	42.49	68.02	45.62	60	45.42	68.02	45.62	77.2
86 - 5608 - 259 ...	86 - 5608 - 260 ...	86 - 5608 - 261 ...	86 - 5608 - 316 ...	-	86 - 5608 - 314 ...	86 - 5608 - 315 ...	150.84	152.5	30.94	32.37	54.36	19.837	29.85	64.14	40.145	54.32	46.32	53.8	46.7	53.8	46.7	54.32	46.32	54.9
86 - 5555 - 259 ...	86 - 5555 - 260 ...	86 - 5555 - 261 ...	86 - 5555 - 316 ...	-	86 - 5555 - 314 ...	86 - 5555 - 315 ...	168.001	150.288	178.399	133.115	92.491	121.257	144.126	97.05	36.16	71.13	43.09	65.45	42.12	55.66	42.18	65.45	42.12	71.1

# Online results submission

- many numerical values

i	z1	z2	z3	z4	z5	z6	z7
	148.33	155.88	202.12	164.35	180.91	30.29	185.19
	25.97	153.5	34.64	35.79	55.56	26.212	10,693
	0	0	0	0	0	0	0
	50	50	50	50	50	50	50



# Online results submission

Grade = Quality of the work +  
+ Quality of the submission

# Examen: Logarithmic scales

$$\text{dB} = 10 \cdot \log_{10} (P_2 / P_1)$$

$$0 \text{ dB} = 1$$

$$+0.1 \text{ dB} = 1.023 (+2.3\%)$$

$$+3 \text{ dB} = 2$$

$$+5 \text{ dB} = 3$$

$$+10 \text{ dB} = 10$$

$$-3 \text{ dB} = 0.5$$

$$-10 \text{ dB} = 0.1$$

$$-20 \text{ dB} = 0.01$$

$$-30 \text{ dB} = 0.001$$

$$\text{dBm} = 10 \cdot \log_{10} (P / 1 \text{ mW})$$

$$0 \text{ dBm} = 1 \text{ mW}$$

$$3 \text{ dBm} = 2 \text{ mW}$$

$$5 \text{ dBm} = 3 \text{ mW}$$

$$10 \text{ dBm} = 10 \text{ mW}$$

$$20 \text{ dBm} = 100 \text{ mW}$$

$$-3 \text{ dBm} = 0.5 \text{ mW}$$

$$-10 \text{ dBm} = 100 \mu\text{W}$$

$$-20 \text{ dBm} = 1 \mu\text{W}$$

$$-30 \text{ dBm} = 1 \text{ nW}$$

$$[\text{dBm}] + [\text{dB}] = [\text{dBm}]$$

$$[\text{dBm}/\text{Hz}] + [\text{dB}] = [\text{dBm}/\text{Hz}]$$

$$[x] + [\text{dB}] = [x]$$

# Computing Loss in circuits

$$\text{LOSS} = \frac{P_{out}}{P_{in}}$$

$$\text{Loss[dB]} = [-] 10 \cdot \log_{10} \left( \frac{P_{out}}{P_{in}} \right)$$

$$\text{Loss[dB]} = [-] 10 \cdot \log_{10} \left( \frac{P_{out}}{P_0} \cdot \frac{P_0}{P_{in}} \right) = [-] 10 \cdot \left[ \log_{10} \left( \frac{P_{out}}{P_0} \right) - \log_{10} \left( \frac{P_{in}}{P_0} \right) \right]$$

$$\text{Loss[dB]} = [-] (P_{out}[\text{dBm}] - P_{in}[\text{dBm}])$$



=

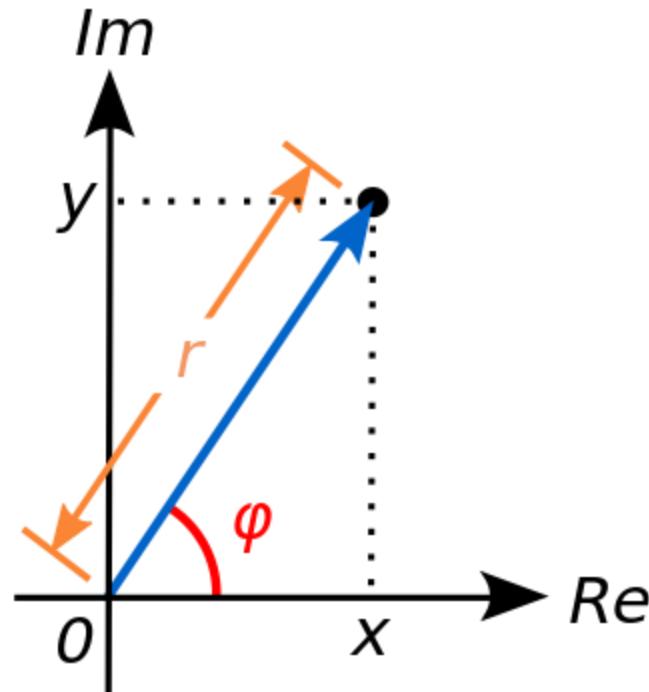


-



# Examen

- Complex numbers arithmetic!!!!
- $z = a + j \cdot b$ ;  $j^2 = -1$

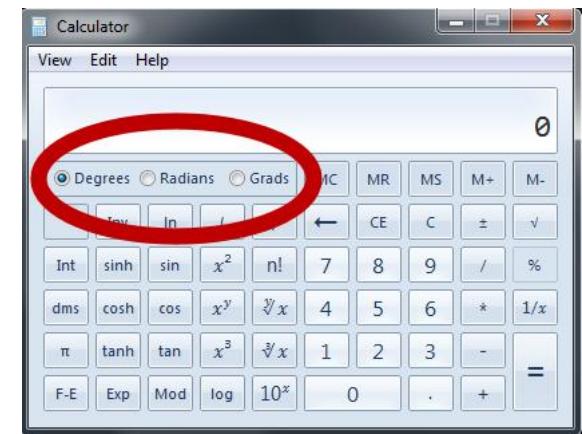


# Polar representation

- **Attention to angle numerical values!!**
  - math software – work in standard unit: radians
    - a **conversion** is necessary before and after using a trigonometric function ( $\sin$ ,  $\cos$ ,  $\tan$ ,  $\text{atan}$ ,  $\tanh$ )
  - scientific calculators have the built-in option of choosing the angle unit
    - always **double check** current working unit

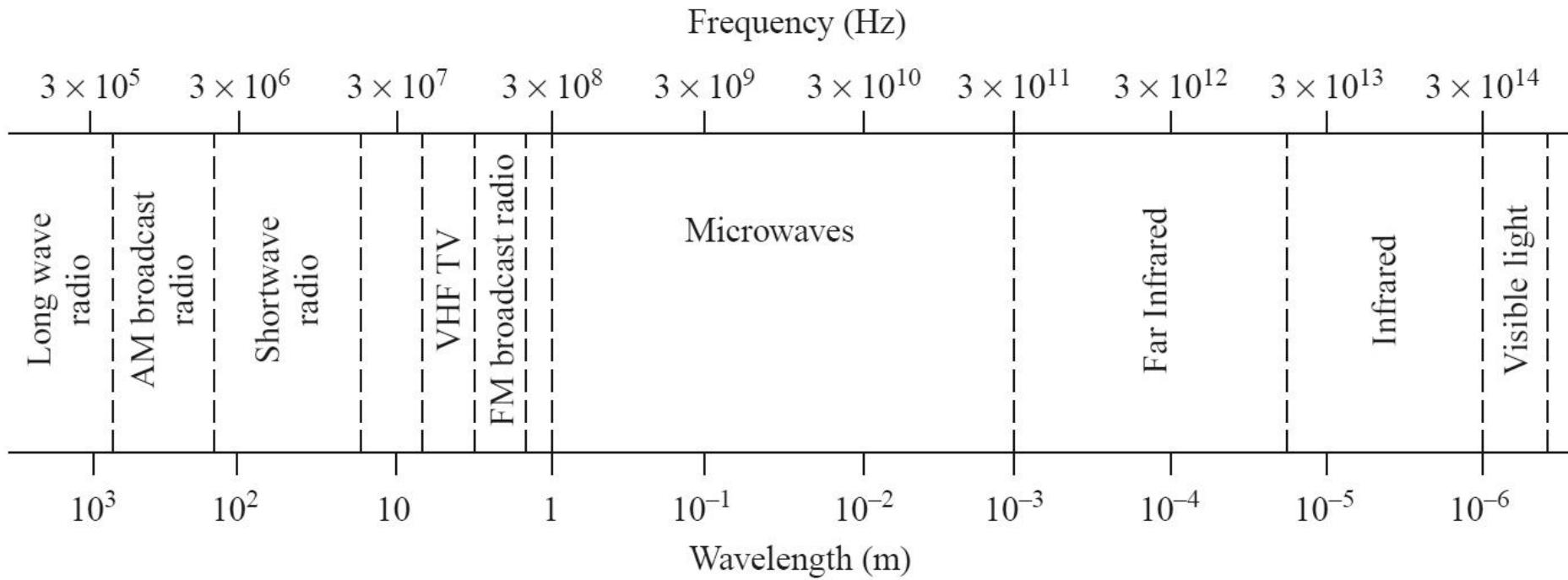
$$\varphi[\circ] = 180^\circ \cdot \frac{\varphi[\text{rad}]}{\pi}$$

$$\varphi[\text{rad}] = \pi \cdot \frac{\varphi[\circ]}{180^\circ}$$



# Introduction

# Microwaves



- typically
  - $f \approx 1\text{--}3\text{GHz} - 300\text{GHz}$
  - $\lambda \approx 1\text{mm} - 10\text{cm}$

# Microwaves

## Typical Frequencies

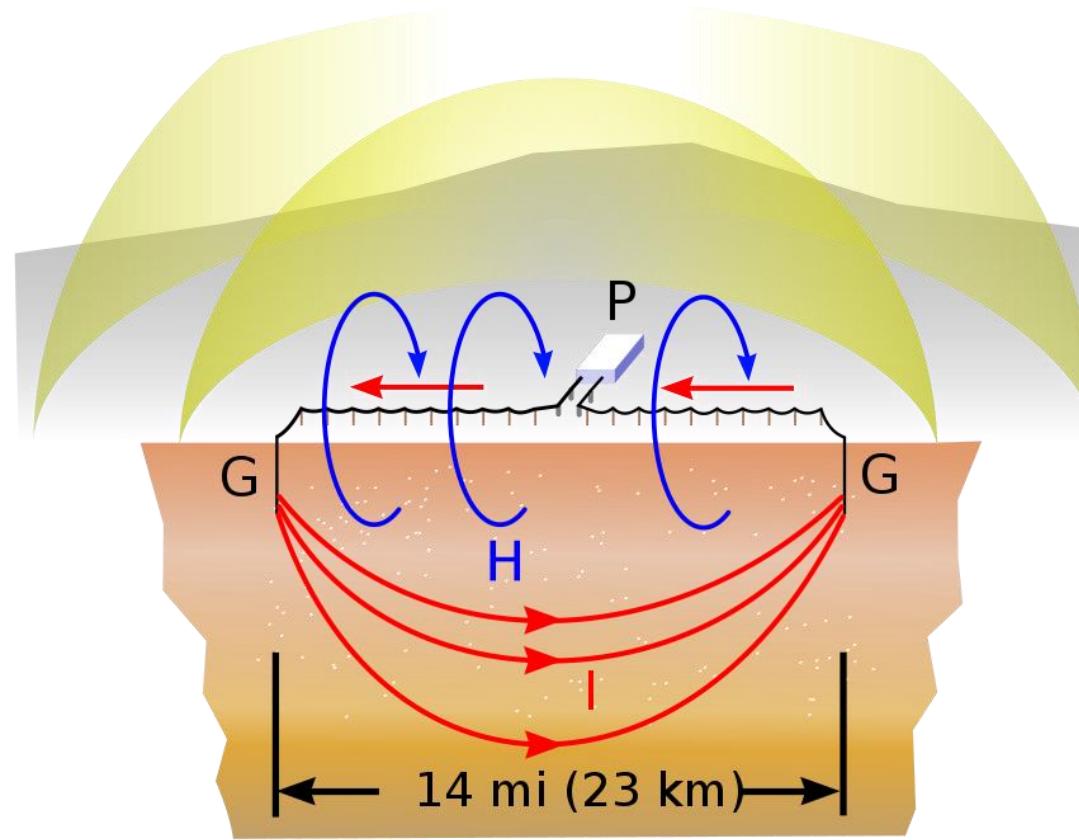
AM broadcast band	535–1605 kHz
Short wave radio band	3–30 MHz
FM broadcast band	88–108 MHz
VHF TV (2–4)	54–72 MHz
VHF TV (5–6)	76–88 MHz
UHF TV (7–13)	174–216 MHz
UHF TV (14–83)	470–890 MHz
US cellular telephone	824–849 MHz 869–894 MHz
European GSM cellular	880–915 MHz 925–960 MHz
GPS	1575.42 MHz 1227.60 MHz
Microwave ovens	2.45 GHz
US DBS	11.7–12.5 GHz
US ISM bands	902–928 MHz 2.400–2.484 GHz 5.725–5.850 GHz
US UWB radio	3.1–10.6 GHz

## Approximate Band Designations

Medium frequency	300 kHz–3 MHz
High frequency (HF)	3 MHz–30 MHz
Very high frequency (VHF)	30 MHz–300 MHz
Ultra high frequency (UHF)	300 MHz–3 GHz
L band	1–2 GHz
S band	2–4 GHz
C band	4–8 GHz
X band	8–12 GHz
Ku band	12–18 GHz
K band	18–26 GHz
Ka band	26–40 GHz
U band	40–60 GHz
V band	50–75 GHz
E band	60–90 GHz
W band	75–110 GHz
F band	90–140 GHz

# ELF, VLF

- Extremely low frequency, 3 - 30 Hz
- Very low frequency, 3 - 30 kHz



# ~ Microwaves

## ■ Electrical Length (Phase Length)

- $I$  – physical length
- $E = \beta \cdot I$  – electrical Length

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left( \frac{l}{\lambda} \right)$$

V, I vary  
~ useless

$$E = \beta \cdot l = \frac{2\pi}{c_0} \cdot \left( l \cdot f \cdot \sqrt{\epsilon_r} \right)$$

## ■ Dependency

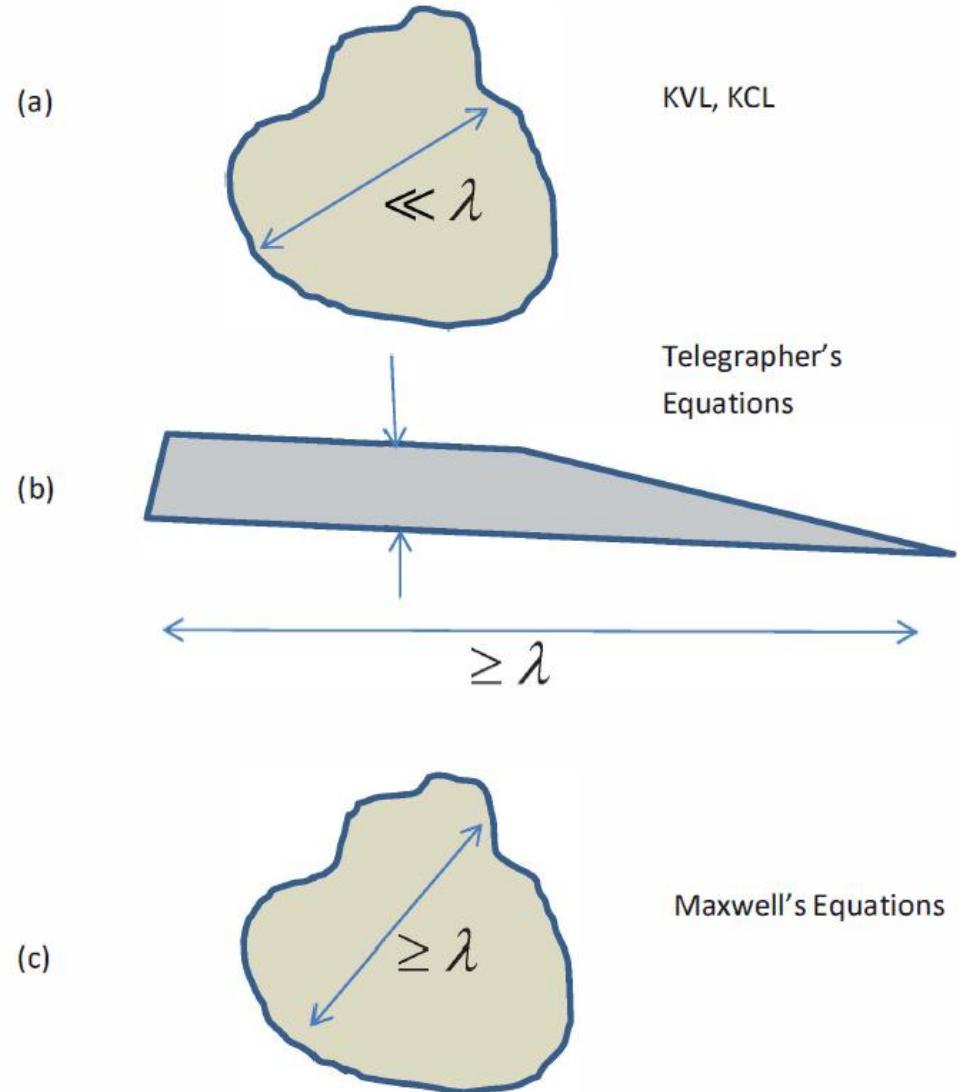
- antenna gain
- radar cross-section

# Electrical Length

- Behavior (and description) of any circuit depends on his electrical length at the particular frequency of interest

- $E \approx 0 \rightarrow$  Kirchhoff
- $E > 0 \rightarrow$  wave propagation

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left( \frac{l}{\lambda} \right)$$



# Maxwell's Equations

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J$$

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

## ■ Constitutive equations

$$D = \epsilon \cdot E$$

$$B = \mu \cdot H$$

$$J = \sigma \cdot E$$

- Vacuum

$$\mu_0 = 4\pi \times 10^{-7} \text{ } H/m$$

$$\epsilon_0 = 8,854 \times 10^{-12} \text{ } F/m$$

$$c_0 = \frac{1}{\sqrt{\epsilon_0 \cdot \mu_0}} = 2,99790 \cdot 10^8 \text{ } m/s$$

# Electromagnetic fields with harmonic time dependence

$$X = X_0 e^{j \cdot \omega \cdot t} \quad \frac{\partial X}{\partial t} = j \cdot \omega \cdot X$$

$$g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \quad f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{j\omega t} d\omega$$

- Maxwell's Equations more simple

$$\nabla^2 E + \omega^2 \epsilon \mu E = j \omega \mu J + \frac{1}{\epsilon} \nabla \rho$$

$$\nabla^2 H + \omega^2 \epsilon \mu H = -\nabla \times J$$

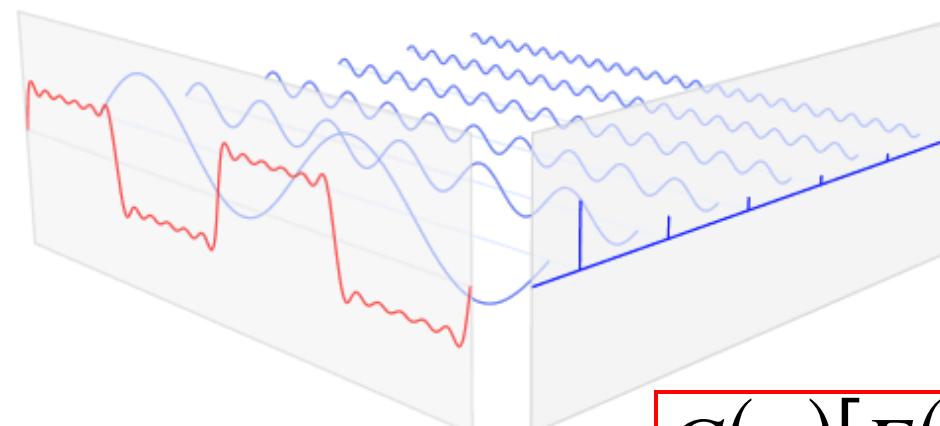
$$\nabla \cdot E = \frac{\rho}{\epsilon}$$

$$\nabla \cdot H = 0$$

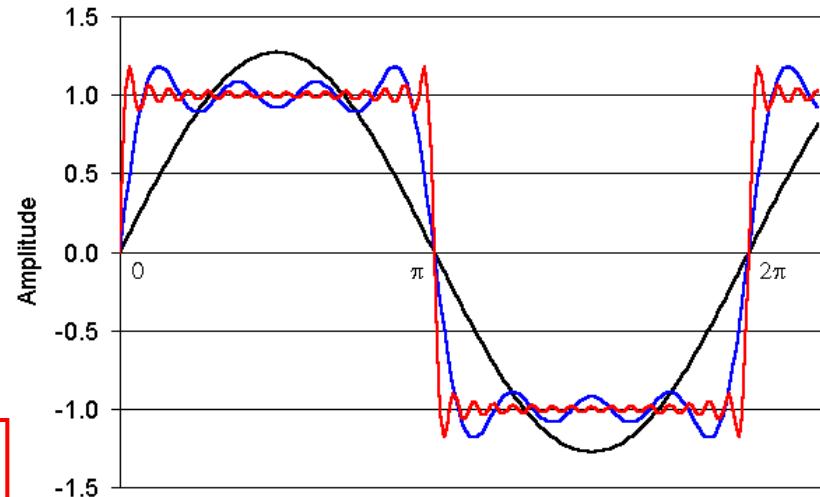
# Mathematical models

- particular cases where analytical solution exists
  - harmonic signals, Fourier Transform, frequency spectrum

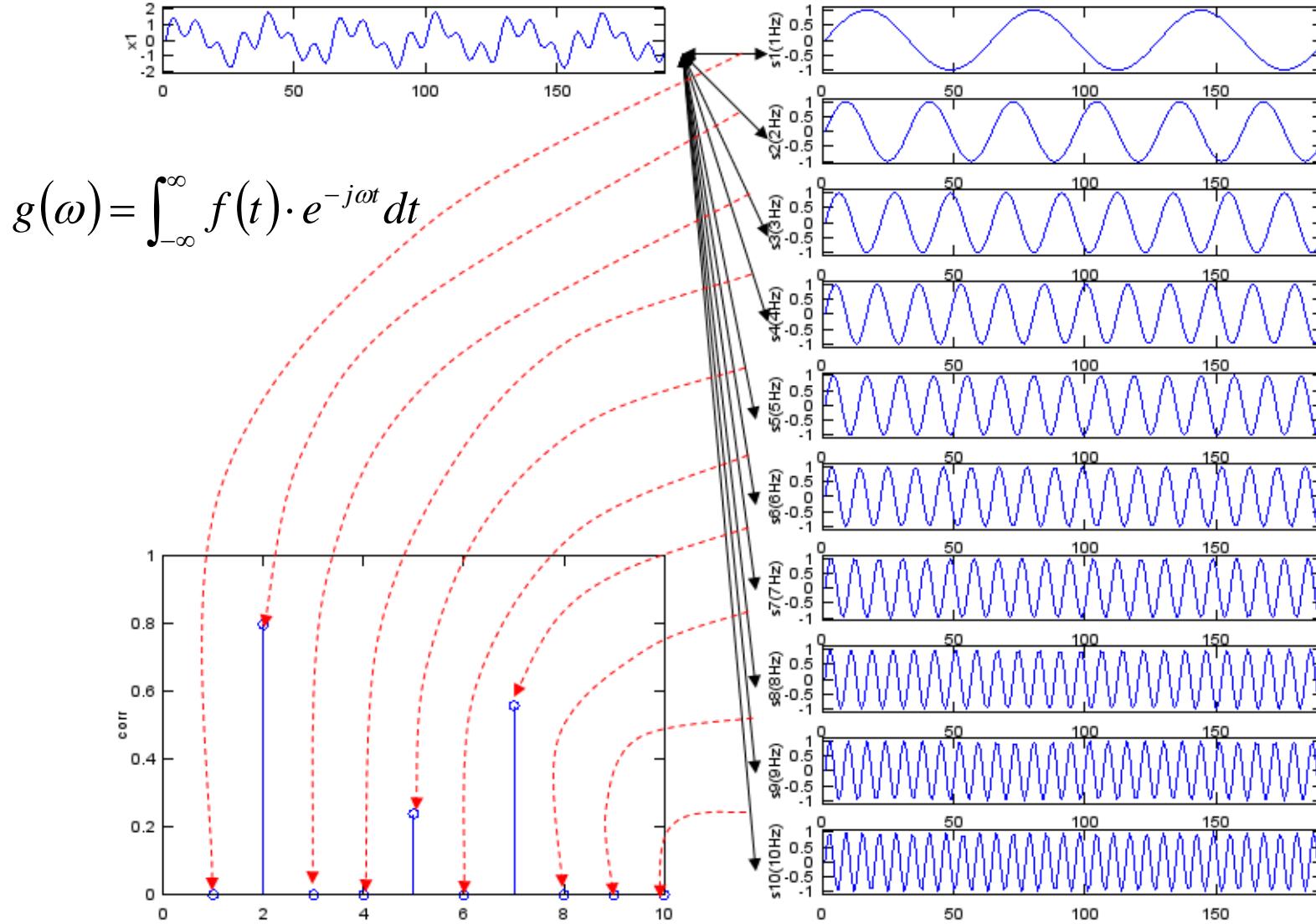
$$X = X_0 e^{j \cdot \omega \cdot t} \quad \frac{\partial X}{\partial t} = j \cdot \omega \cdot X \quad g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \quad f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{j\omega t} d\omega$$



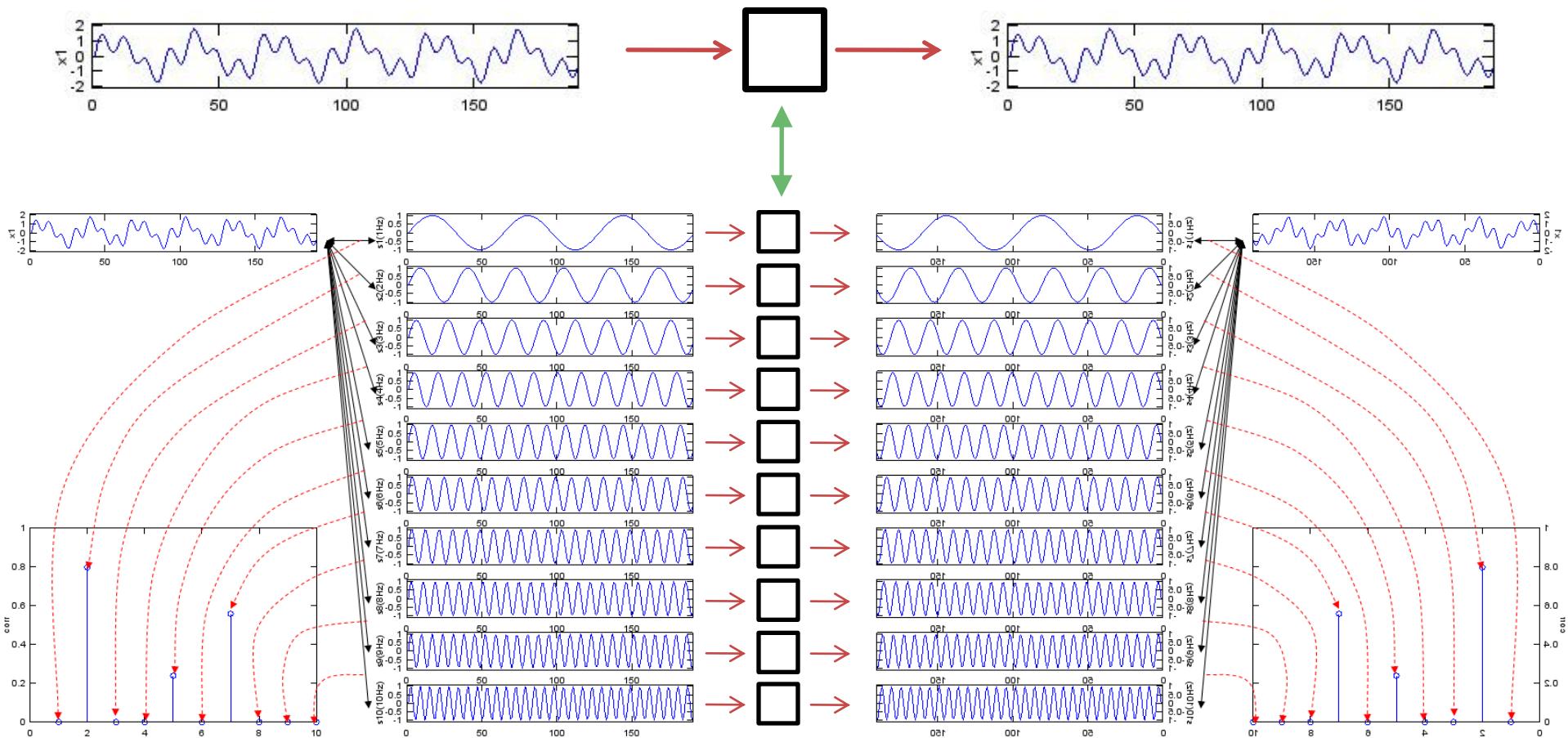
$$G(\omega)[F(\omega)]$$



# Mathematical models



# Mathematical models



$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt$$

$$G(\omega)[F(\omega)]$$

$$g(t) = \int_{-\infty}^{\infty} G(\omega) \cdot e^{j\omega t} d\omega$$

# Wave equations

- Helmholtz equations or Wave equations

Medium void of free electric charges

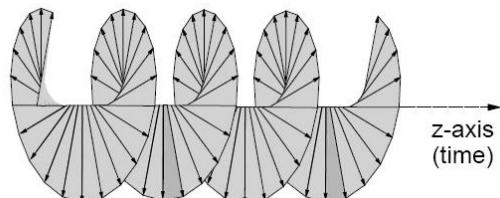
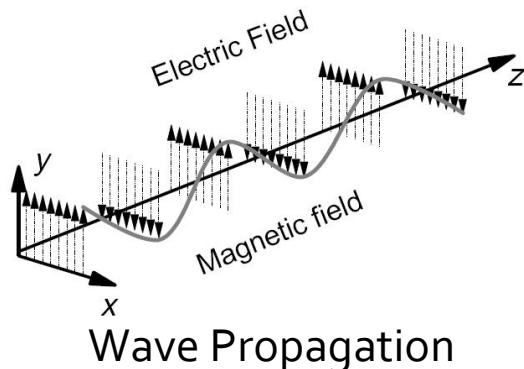
$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

$$\gamma^2 = -\omega^2 \epsilon \mu + j \omega \mu \sigma$$

$\gamma$  – propagation constant (known also as phase constant or wave number)

# Solutions of the wave equations



Circular Polarization

Electric field only in Oy direction, ← through judicious choice  
wave traveling after Oz direction ← of the coordinate system

$$E_y = E_+ e^{-\gamma \cdot z} + E_- e^{\gamma \cdot z}$$

$$\gamma = \sqrt{-\omega^2 \epsilon \mu + j \omega \mu \sigma} = \alpha + j \cdot \beta$$

If we have only the positive direction wave  $E_+ \Rightarrow A$

$$E_y = A e^{-(\alpha + j \cdot \beta) \cdot z}$$

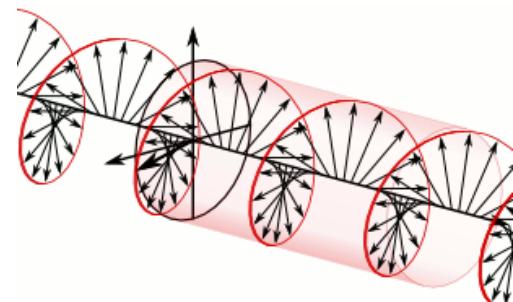
Harmonic Field

$$E_y = A \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

Amplitude

Attenuation

Wave Propagation  
(simultaneous space and time variation)



# Attenuation

$$E_y(z_1) = Ct \cdot e^{-\alpha \cdot z_1} \cdot e^{j(\omega \cdot t - \beta \cdot z_1)}$$

$$E_y(z_2) = Ct \cdot e^{-\alpha \cdot z_2} \cdot e^{j(\omega \cdot t - \beta \cdot z_2)}$$

$$W, P \sim \int E^2$$

$$A = \frac{P_2}{P_1} = \frac{Ct^2 \cdot e^{-2\alpha \cdot z_2}}{Ct^2 \cdot e^{-2\alpha \cdot z_1}} = e^{-2\alpha \cdot (z_2 - z_1)}$$

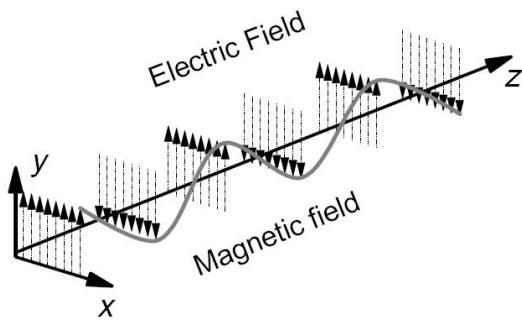
$$A[dB] = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} [e^{-2\alpha \cdot (z_2 - z_1)}]$$

$$A[dB] = -20 \cdot \alpha \cdot (z_2 - z_1) \log_{10} e = -8.686 \cdot \alpha \cdot (z_2 - z_1)$$

$$A / L[dB / km] = -8.686 \cdot \alpha < 0$$

- ▶ Attenuation usually expressed in **dB/km**
  - ▶ most of the time a positive value is used
  - ▶ “-” sign = **implied** by the word used

# Plane wave parameters



$$\nabla \times E = -j\omega\mu \cdot H$$

$$H_x = \frac{j\gamma \cdot E_y}{\omega\mu}$$

Lossless Medium,  $\sigma = 0$

$$\gamma = j\omega \cdot \sqrt{\epsilon\mu}$$

$$\eta = \frac{E_y}{H_x} = \sqrt{\frac{\mu}{\epsilon}} \quad \text{intrinsic impedance of the medium}$$

$$E_y = A \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

constant phase points:  $(\omega \cdot t - \beta \cdot z) = \text{const}$

Phase velocity

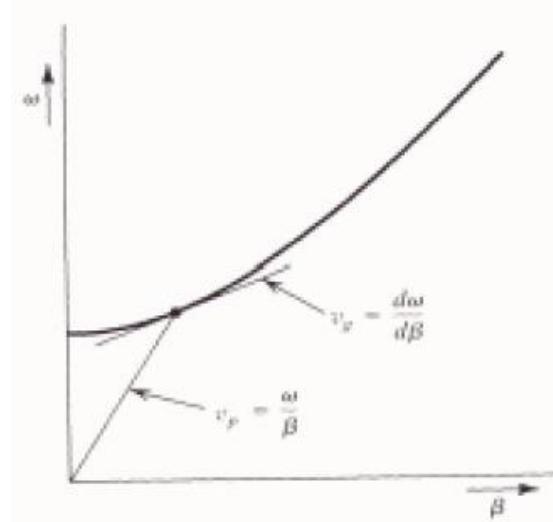
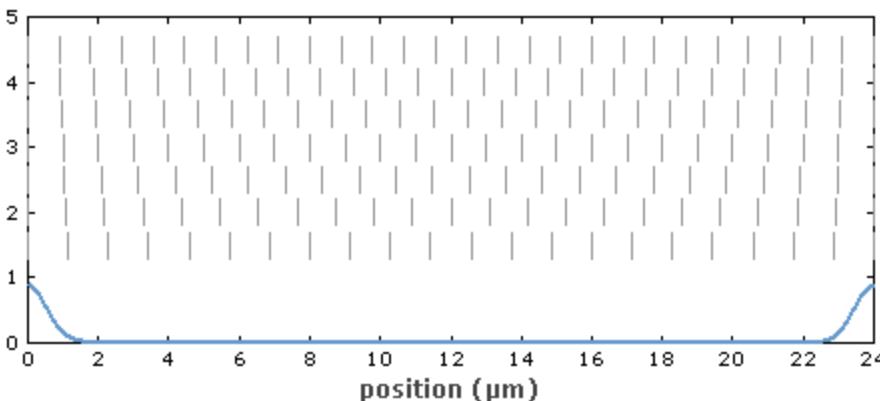
$$v_p = \frac{dz}{dt} = \frac{\omega}{\beta} = \frac{1}{\sqrt{\epsilon\mu}}$$

Group velocity

$$v_g = \frac{dz}{dt} = \frac{d\omega}{d\beta} \quad \text{in dispersive media where } \beta = \beta(\omega)$$

# Group and phase velocities

- Phase velocity – **virtual** speed at which a constant phase point travels (in certain conditions might be greater than the speed of light)
- Group velocity – speed at which the signal (energy, information) propagates (always less or equal to the speed of light in that medium)



# Plane wave parameters

## ■ In vacuum

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \Omega \quad v = v_g = c_0 \quad c_0 = \frac{1}{\sqrt{\epsilon_0 \cdot \mu_0}} = 2,99790 \cdot 10^8 \text{ m/s}$$

$$\lambda_0 = \frac{2\pi}{\beta} = \frac{c_0}{f} \quad T = \frac{2\pi}{\omega} = \frac{1}{f}$$

Space periodicity

Time periodicity

## ■ In non-dispersive medium with $\epsilon_r$

$$c = \frac{1}{\sqrt{\epsilon \cdot \mu_0}} = \frac{1}{\sqrt{\epsilon_0 \epsilon_r \cdot \mu_0}} = \frac{c_0}{\sqrt{\epsilon_r}}$$

$$n = \sqrt{\epsilon_r} \quad \text{refractive index of a medium}$$

$$c = \frac{c_0}{n}$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f}$$

$$\lambda = \frac{c_0}{\sqrt{\epsilon_r \cdot f}} = \frac{\lambda_0}{\sqrt{\epsilon_r}}$$



# Solutions of the wave equations

$$E_y = E^+ e^{-\gamma \cdot z} + E^- e^{\gamma \cdot z}$$

Electric field only in Oy direction, ← through judicious choice  
wave traveling after Oz direction ← of the coordinate system

$$\gamma = \sqrt{-\omega^2 \epsilon \mu + j \omega \mu \sigma} = \alpha + j \cdot \beta$$

- wave
  - incident
  - reflected
- wave
  - direct
  - inverse

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

$$(\omega \cdot t - \beta \cdot z) = \text{const}$$

$$E_y = E^- \cdot e^{\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$(\omega \cdot t + \beta \cdot z) = \text{const}$$

points of  
constant  
phase

# Solutions of the wave equations

## ■ wave

- incident
- reflected

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + E^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$H_z = H^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + H^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

## ■ wave

- direct
- inverse

$$V(z) = V^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + V^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$I(z) = I^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + I^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$V(z) = V^+ \cdot e^{j(\omega \cdot t - \beta \cdot z)} + V^- \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

# Mathematical modeling

- particular cases where analytical solution exists

- wave in a single direction  $E^+(E^+)$ ,  $E^-(E^-)$

- wave

- incident

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + E^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

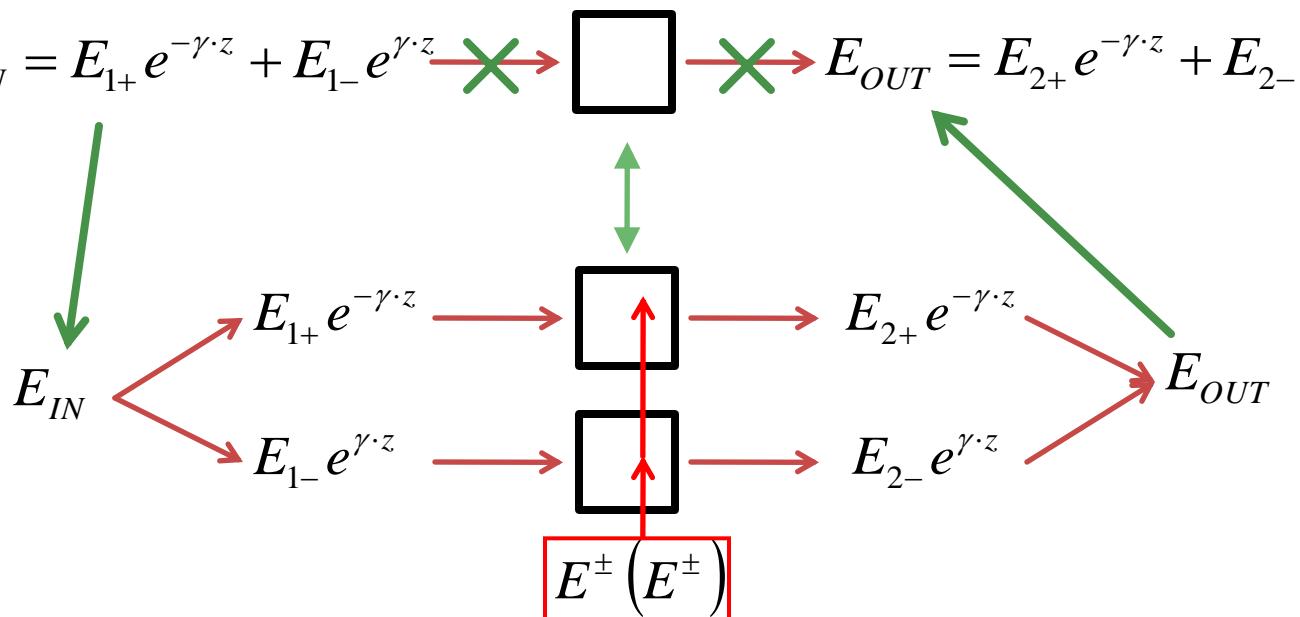
- reflected

$$E_{IN} = E_{1+} e^{-\gamma \cdot z} + E_{1-} e^{\gamma \cdot z} \quad \text{---} \rightarrow \boxed{\text{ }} \quad \text{---} \rightarrow E_{OUT} = E_{2+} e^{-\gamma \cdot z} + E_{2-} e^{\gamma \cdot z}$$

- wave

- direct

- inverse



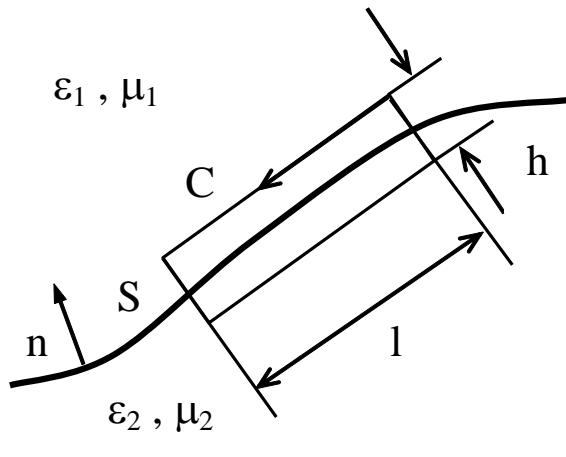
# Modes in delimited media

- Electromagnetic fields with harmonic time dependence
  - Maxwell's Equations simplified

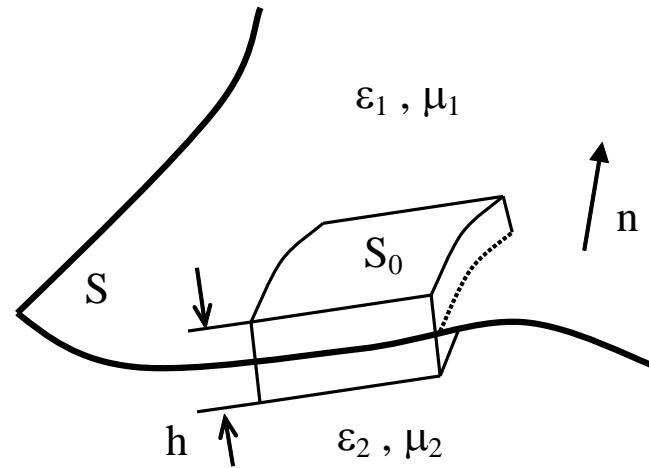
$$X = X_0 e^{j\omega t} \quad \frac{\partial X}{\partial t} = j \cdot \omega \cdot X \quad g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \quad f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{j\omega t} d\omega$$

- In delimited media the solutions of Maxwell's Equations must also verify boundary conditions
  - solutions must respect some supplemental conditions

# Interface conditions on the interface between two different media



a)



b)

$$n \times (E_1 - E_2) = 0$$

$$n \cdot (D_1 - D_2) = \rho_s$$

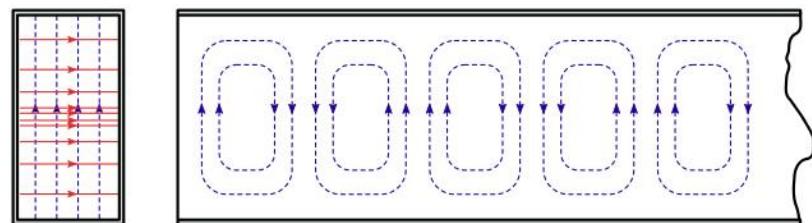
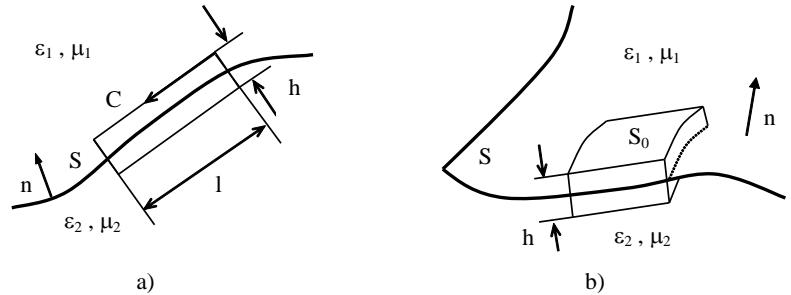
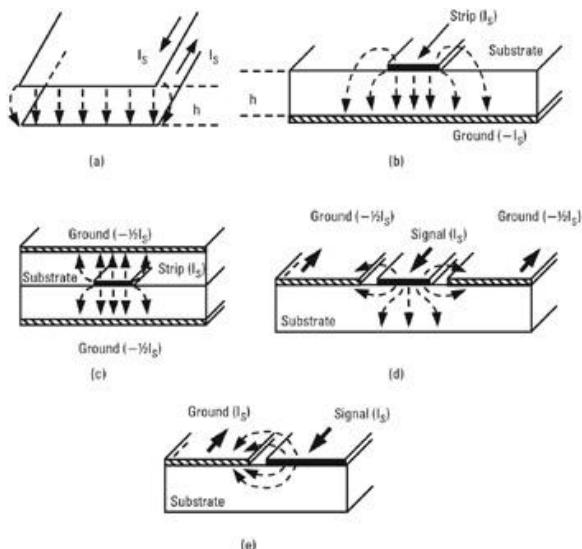
$$n \times (H_1 - H_2) = J_s$$

$$n \cdot (B_1 - B_2) = 0$$

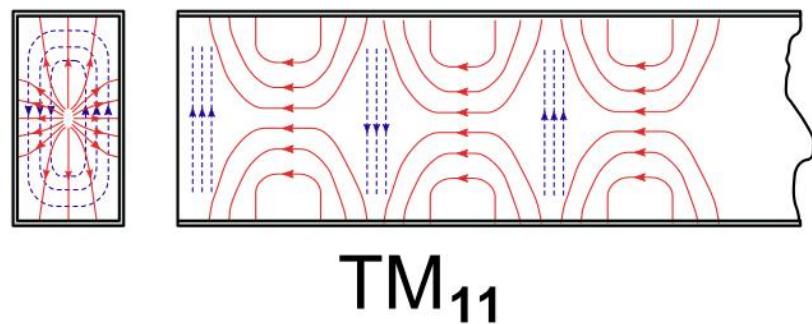
- If one of the media is a perfect conductor (metal) all fields are annulled inside

# Modes in delimited media

- Electric field must always be normal on an electric wall or annulled
- Magnetic field must always be tangent to an electric wall or annulled

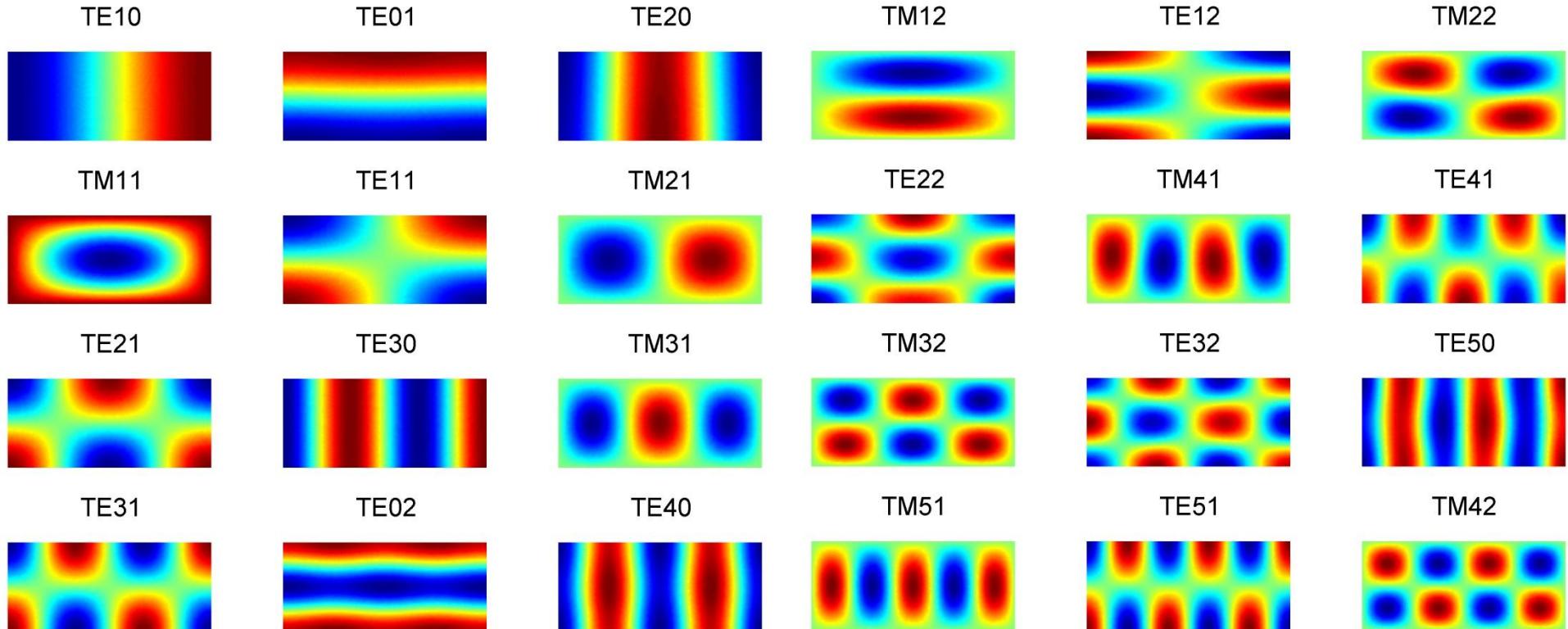


**TE<sub>10</sub>**



**TM<sub>11</sub>**

# Moduri in mediis delimitate



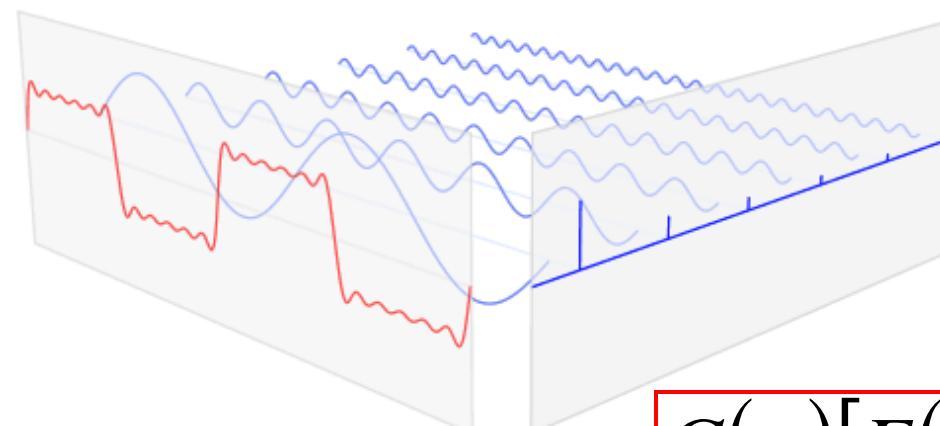
- Similar with Fourier Transform 
$$g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \quad f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{j\omega t} d\omega$$

$$E^+, E^- = \sum_1^{\infty} A_i \cdot Mod_i \quad A_i = \langle E, Mod_i \rangle$$

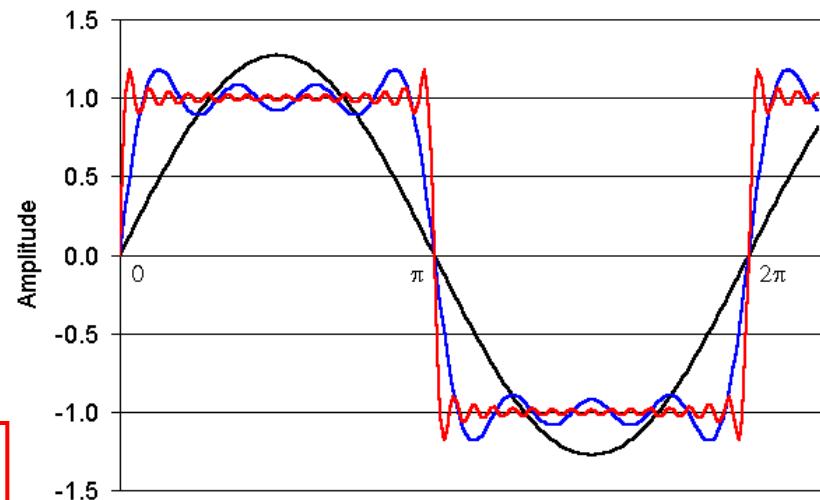
# Mathematical modeling

- particular cases where analytical solution exists
  - harmonic signals, Fourier Transform, frequency spectrum

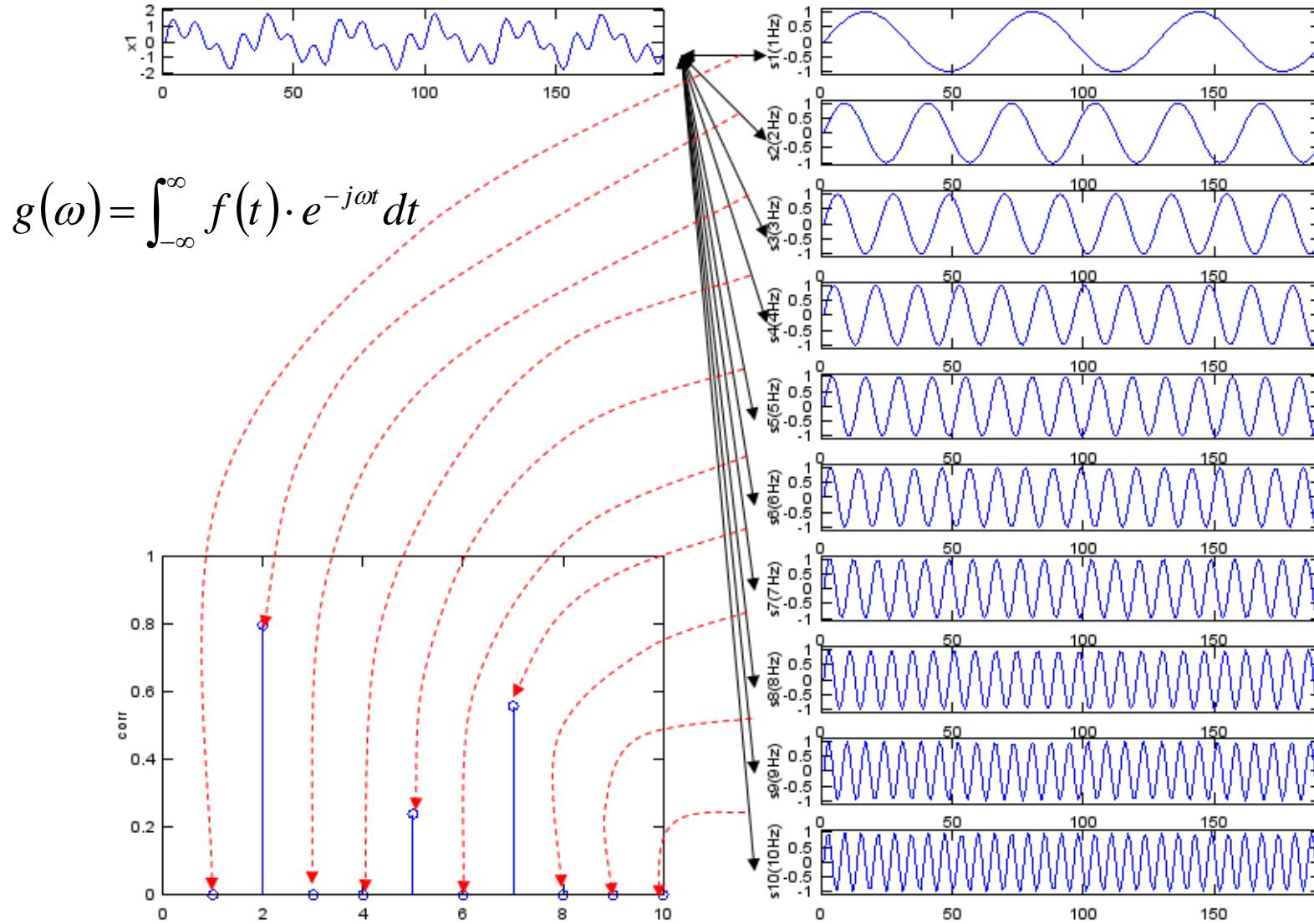
$$X = X_0 e^{j \cdot \omega \cdot t} \quad \frac{\partial X}{\partial t} = j \cdot \omega \cdot X \quad g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \quad f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{j\omega t} d\omega$$



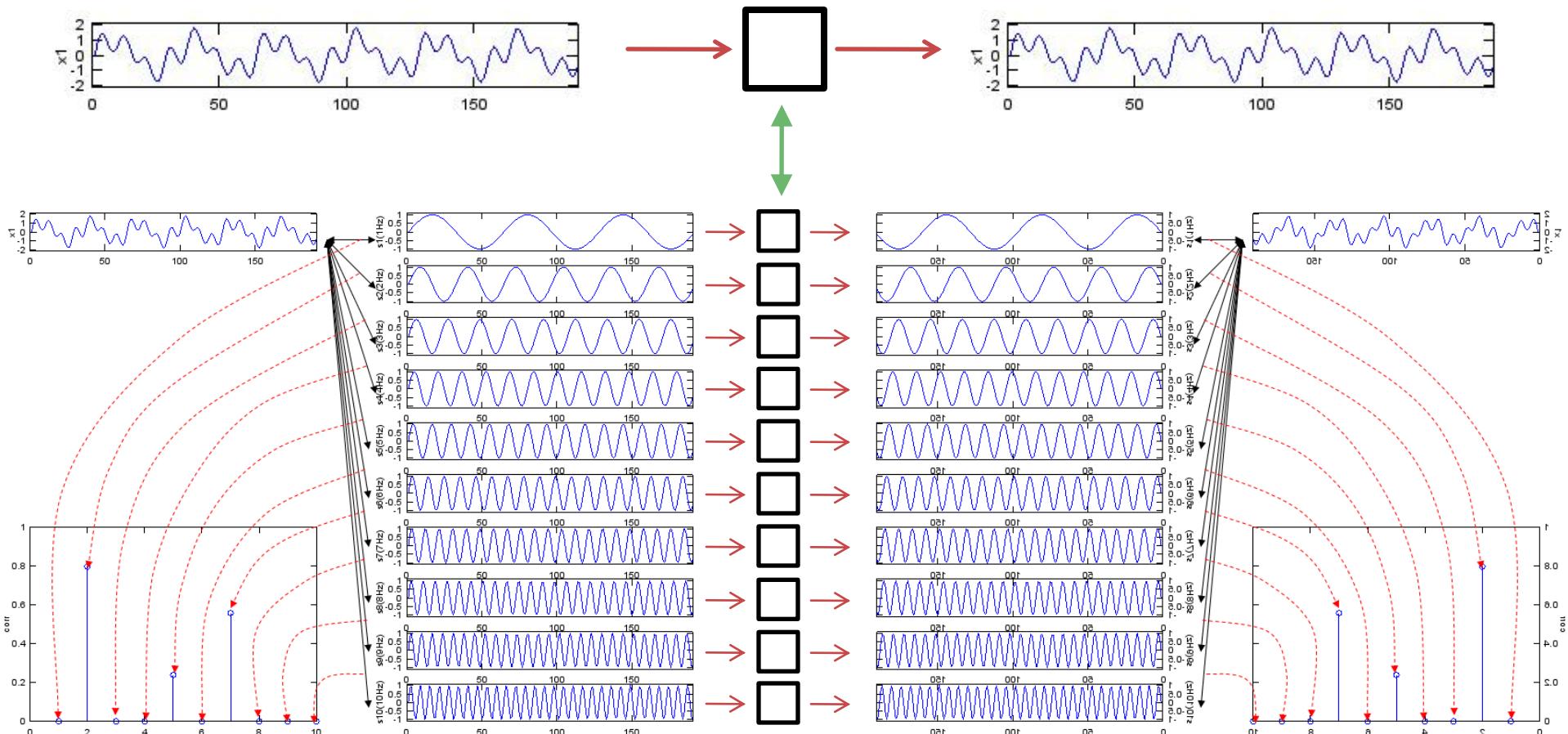
$$G(\omega)[F(\omega)]$$



# Mathematical modeling



# Mathematical modeling



$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt$$

$$G(\omega)[F(\omega)]$$

$$g(t) = \int_{-\infty}^{\infty} G(\omega) \cdot e^{j\omega t} d\omega$$

# Mathematical modeling

- particular cases where analytical solution exists

- wave in a single direction  $E^+(E^+)$ ,  $E^-(E^-)$

- wave

- incident

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + E^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

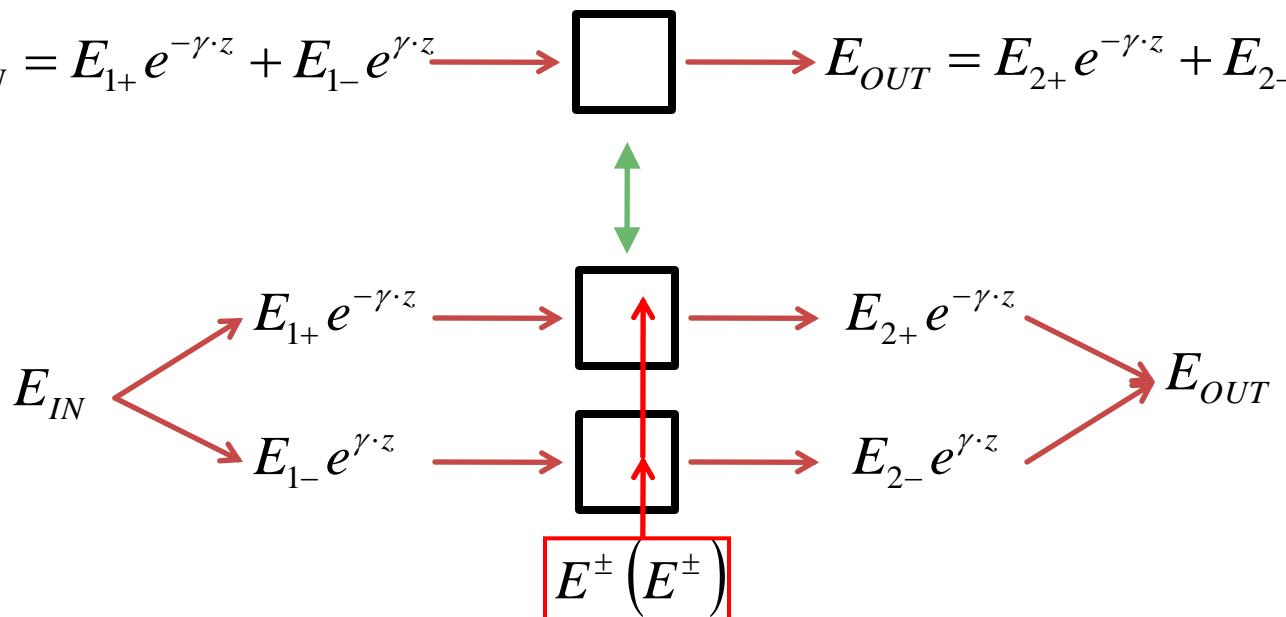
- reflected

$$E_{IN} = E_{1+} e^{-\gamma \cdot z} + E_{1-} e^{\gamma \cdot z} \rightarrow \boxed{\text{ }} \rightarrow E_{OUT} = E_{2+} e^{-\gamma \cdot z} + E_{2-} e^{\gamma \cdot z}$$

- wave

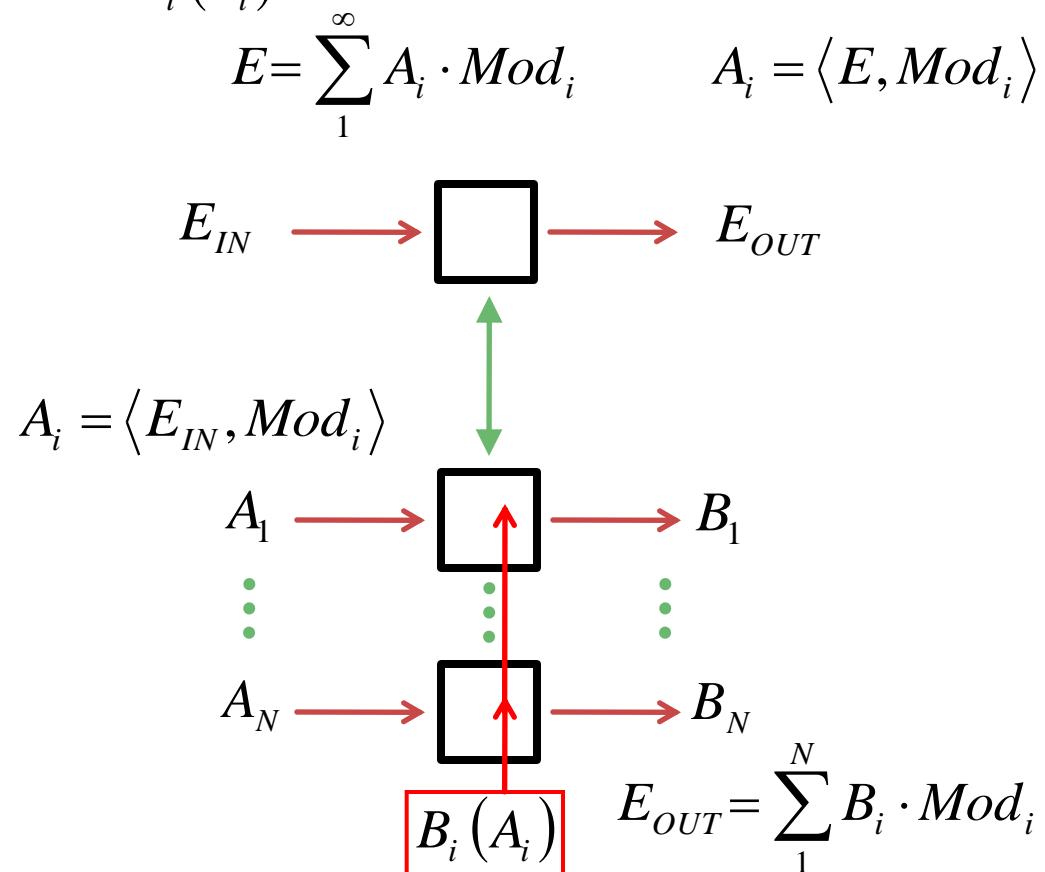
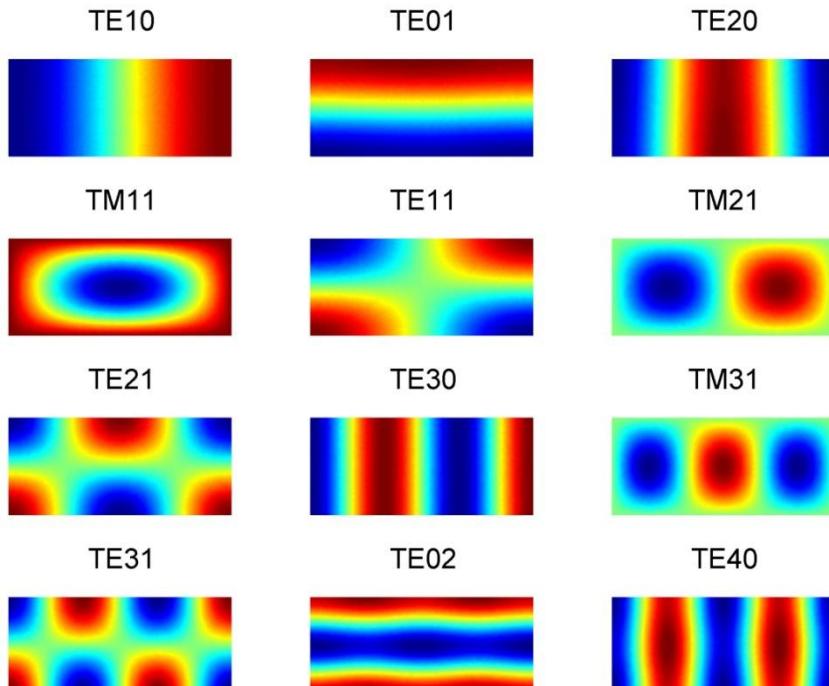
- direct

- inverse



# Mathematical modeling

- particular cases where analytical solution exists
  - modes in delimited media  $B_i(A_i)$



# TEM transmission lines

# Course Topics

- **Transmission lines**
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers?~~

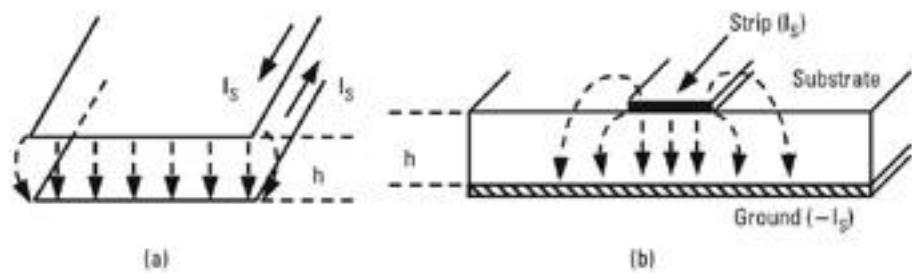
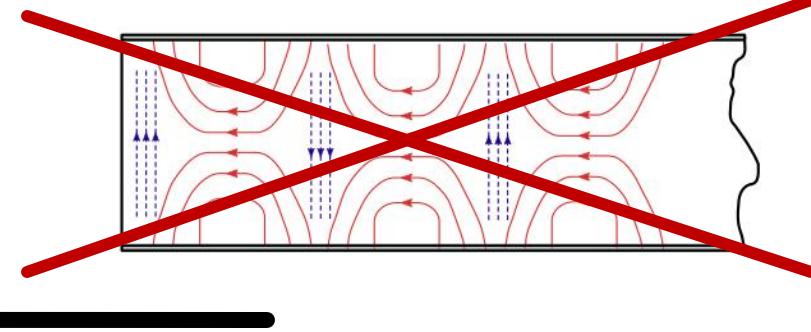
# Transmission line

- TEM wave propagation, at least two conductors

$$I(z, t)$$



$$V(z, t)$$

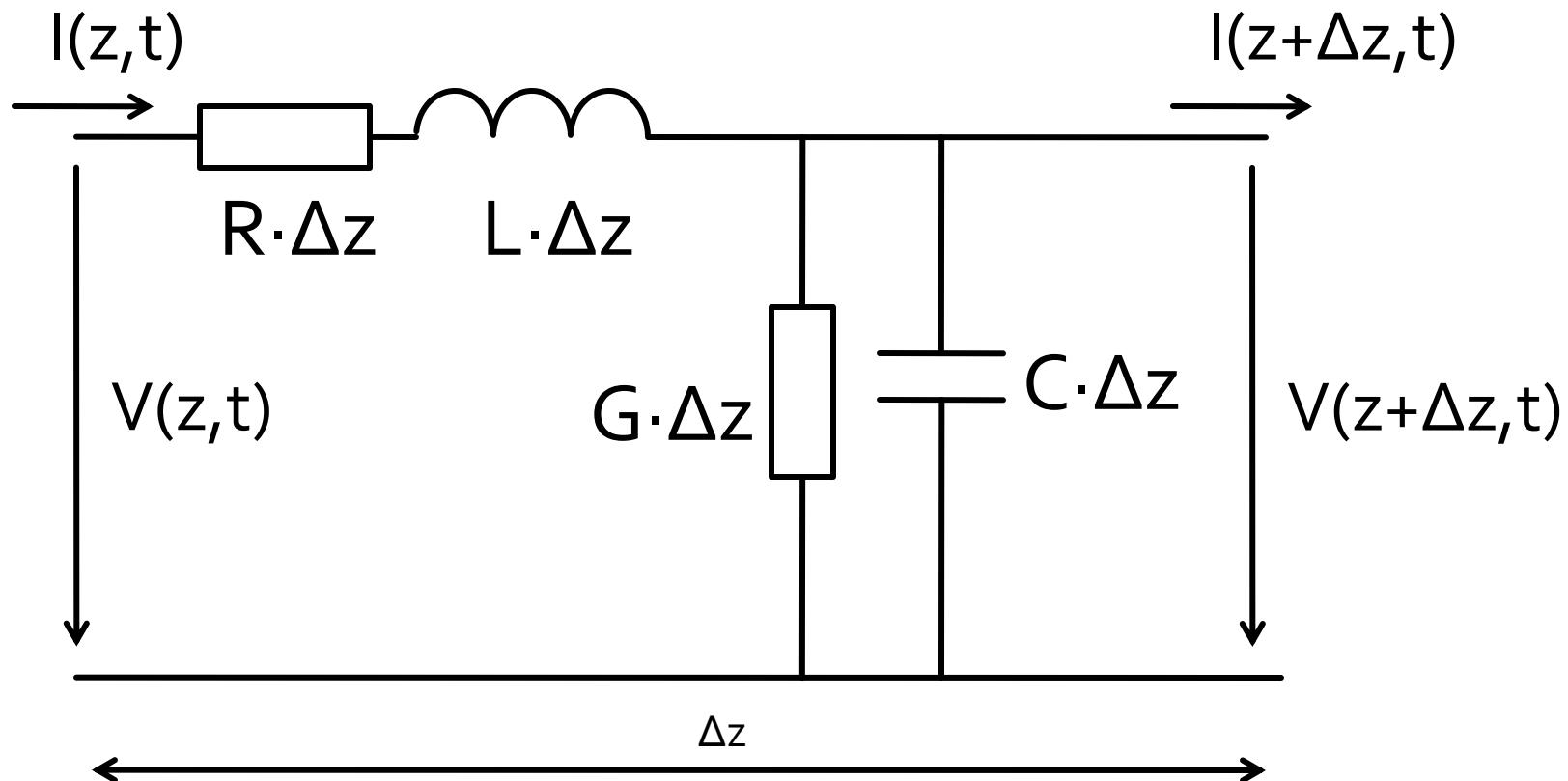


(a)

(b)

# Transmission line equivalent model

- TEM wave propagation, at least two conductors



- distributed** (line) parameters  $R, L, G, C$  (eg.  $\Omega/m$ )

# Telegrapher's equations

- time domain

$$\frac{\partial v(z,t)}{\partial z} = -R \cdot i(z,t) - L \cdot \frac{\partial i(z,t)}{\partial t} \quad K \parallel$$

$$\frac{\partial i(z,t)}{\partial z} = -G \cdot v(z,t) - C \cdot \frac{\partial v(z,t)}{\partial t} \quad K \perp$$

- harmonic signals (frequency domain)

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z) \quad \left/ \frac{d}{dz} (\dots) \right.$$

$$\frac{dI(z)}{dz} = -(G + j \cdot \omega \cdot C) \cdot V(z)$$

# Solving T's E

$$\frac{d^2V(z)}{dz^2} - \gamma^2 \cdot V(z) = 0$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 \cdot I(z) = 0$$



$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

$$\gamma^2 = -\omega^2 \epsilon \mu + j \omega \mu \sigma$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$E_y = E_+ e^{-\gamma \cdot z} + E_- e^{\gamma \cdot z}$$

# Solutions

$$\left\{ \begin{array}{l} V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \end{array} \right.$$

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$Z_0 \equiv \frac{R + j \cdot \omega \cdot L}{\gamma} = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}}$$

$$\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$I(z) = \frac{\gamma}{R + j \cdot \omega \cdot L} (V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z})$$

- Characteristic impedance of the line

$$\lambda = \frac{2\pi}{\beta} \quad v_f = \frac{\omega}{\beta} = \lambda \cdot f$$

# The lossless line

- **Lossless:**  $R=G=0$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)} = j \cdot \omega \cdot \sqrt{L \cdot C}$$

$$\alpha = 0 \quad ; \quad \beta = \omega \cdot \sqrt{L \cdot C}$$

$$Z_0 = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}} = \sqrt{\frac{L}{C}}$$

- $Z_0$  is **real**

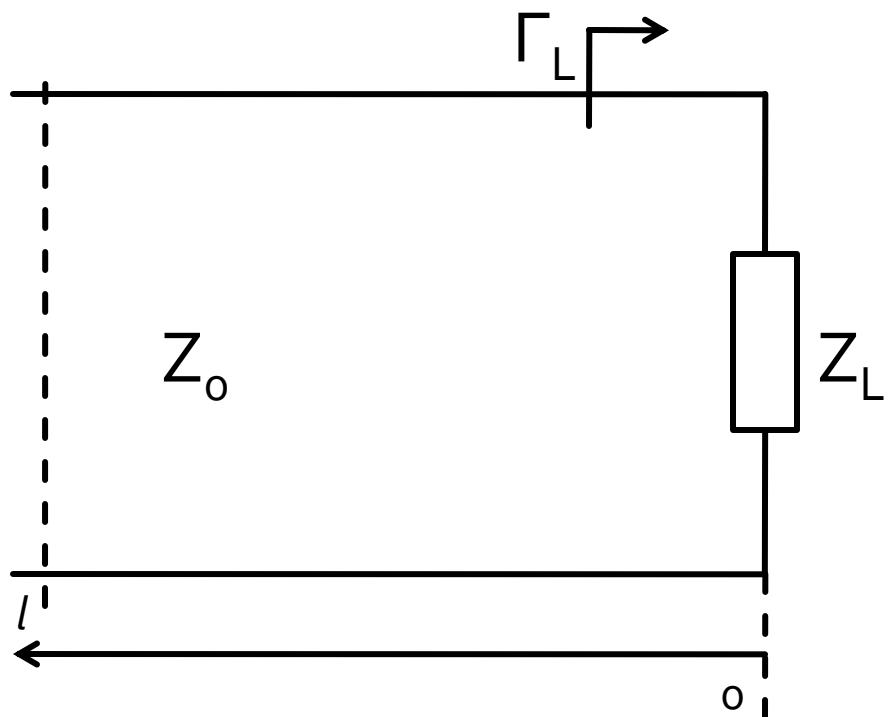
$$V(z) = V_0^+ e^{-j \cdot \beta \cdot z} + V_0^- e^{j \cdot \beta \cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j \cdot \beta \cdot z} - \frac{V_0^-}{Z_0} e^{j \cdot \beta \cdot z}$$

$$\lambda = \frac{2\pi}{\omega \cdot \sqrt{LC}}$$

$$v_f = \frac{1}{\sqrt{LC}}$$

# The lossless line



$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

$$Z_L = \frac{V(0)}{I(0)} \quad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

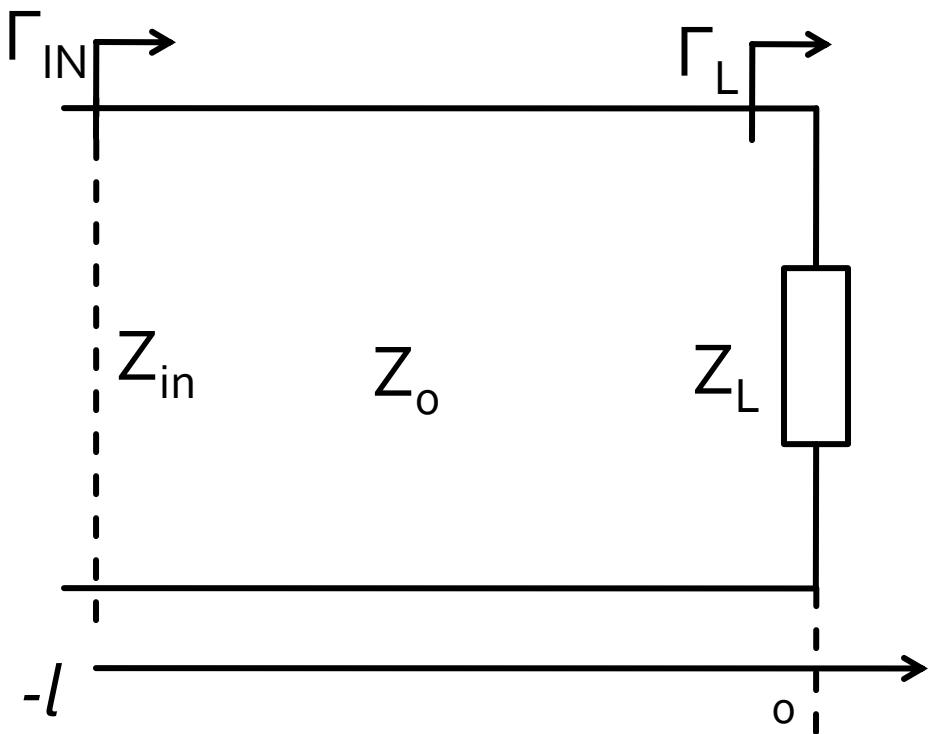
- $Z_0$  real

# The lossless line

- voltage reflection coefficient seen at the input of the line

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$\Gamma = \Gamma(z) = \frac{V_0^-(z)}{V_0^+(z)}$$



$$V(0) = V_0^+ + V_0^-$$

$$\Gamma(0) = \Gamma_L = \frac{V_0^-}{V_0^+}$$

$$V(-l) = V_0^+ e^{j\beta l} + V_0^- e^{-j\beta l}$$

$$\Gamma(-l) = \Gamma_{IN} = \frac{V_0^- \cdot e^{-j\beta l}}{V_0^+ \cdot e^{j\beta l}} = \Gamma(0) \cdot e^{-2j\beta l}$$

$$|\Gamma(-l)| = |\Gamma(0)| \cdot |e^{-2j\beta l}| = |\Gamma(0)|$$

$$\boxed{\Gamma_{IN} = \Gamma_L \cdot e^{-2j\beta l}}$$

$$\boxed{|\Gamma_{IN}| = |\Gamma_L|}$$

# The lossless line

$$V(z) = V_0^+ \cdot (e^{-j\beta z} + \Gamma \cdot e^{j\beta z})$$

$$I(z) = \frac{V_0^+}{Z_0} \cdot (e^{-j\beta z} - \Gamma \cdot e^{j\beta z})$$

- time-average Power flow along the line

$$P_{avg} = \frac{1}{2} \cdot \text{Re}\{V(z) \cdot I(z)^*\} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot \text{Re}\left\{1 - \Gamma^* \cdot e^{-2j\beta z} + \Gamma \cdot e^{2j\beta z} - |\Gamma|^2\right\}$$

$$P_{avg} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot (1 - |\Gamma|^2)$$

$$(z - z^*) = \text{Im}$$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB] 
$$\text{RL} = -20 \cdot \log|\Gamma| \quad [\text{dB}]$$

# Polar representation

- Euler's formula

$$e^{j \cdot x} = \cos x + j \cdot \sin x; \forall x \in R$$

- Polar representation

$$z = a + j \cdot b = |z| \cdot e^{j \cdot \varphi}$$

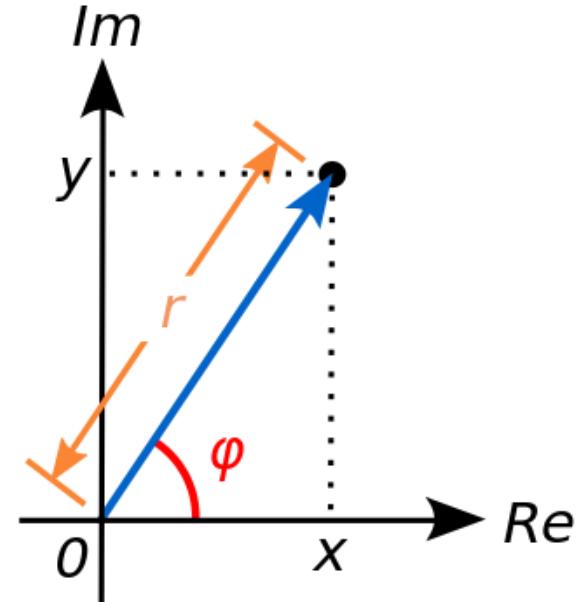
$$z = a + j \cdot b = |z| \cdot (\cos \varphi + j \cdot \sin \varphi)$$

$$z^n = (|z| \cdot e^{j \cdot \varphi})^n = |z|^n \cdot e^{j \cdot n \cdot \varphi} = |z|^n \cdot [\cos(n \cdot \varphi) + j \cdot \sin(n \cdot \varphi)]$$

$$\sqrt{z} = (|z| \cdot e^{j \cdot \varphi})^{1/2} = \sqrt{|z|} \cdot e^{j \cdot \frac{\varphi}{2}} = \sqrt{|z|} \cdot \left( \cos \frac{\varphi}{2} + j \cdot \sin \frac{\varphi}{2} \right)$$

$$z \cdot w = |z| \cdot e^{j \cdot \varphi} \cdot |w| \cdot e^{j \cdot \theta} = |z| \cdot |w| \cdot e^{j \cdot (\varphi + \theta)} = |z| \cdot |w| \cdot [\cos(\varphi + \theta) + j \cdot \sin(\varphi + \theta)]$$

$$z/w = \frac{|z| \cdot e^{j \cdot \varphi}}{|w| \cdot e^{j \cdot \theta}} = \frac{|z|}{|w|} \cdot e^{j \cdot \varphi} \cdot e^{-j \cdot \theta} = \frac{|z|}{|w|} \cdot [\cos(\varphi - \theta) + j \cdot \sin(\varphi - \theta)]$$



# Polar representation

## ■ Euler's formula

$$e^{j \cdot x} = \cos x + j \cdot \sin x; \forall x \in R$$

$$e^{j \cdot x} + e^{-j \cdot x} = \cos x + j \cdot \sin x + \cos(-x) + j \cdot \sin(-x)$$

$$e^{j \cdot x} + e^{-j \cdot x} = \cos x + j \cdot \sin x + \cos x - j \cdot \sin x = 2 \cdot \cos x$$

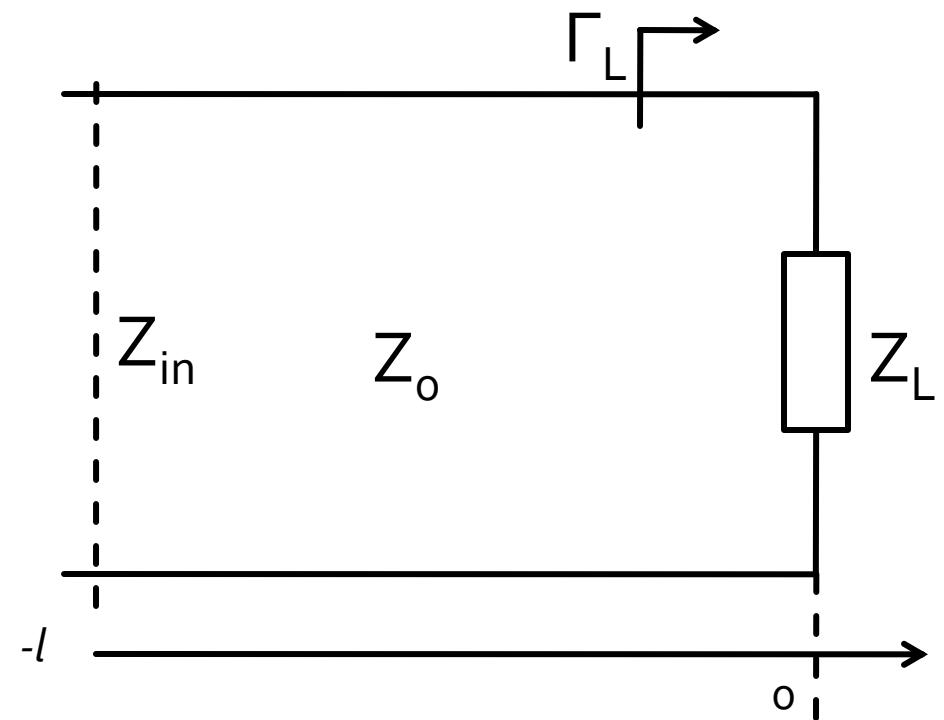

$$\cos x = \frac{e^{j \cdot x} + e^{-j \cdot x}}{2}$$

$$e^{j \cdot x} - e^{-j \cdot x} = \cos x + j \cdot \sin x - \cos(-x) - j \cdot \sin(-x)$$

$$e^{j \cdot x} - e^{-j \cdot x} = \cos x + j \cdot \sin x - \cos x + j \cdot \sin x = 2j \cdot \sin x$$


$$\sin x = \frac{e^{j \cdot x} - e^{-j \cdot x}}{2j}$$

# The lossless line



$$V(-l) = V_0^+ e^{j \cdot \beta \cdot l} + V_0^- e^{-j \cdot \beta \cdot l}$$

$$I(-l) = \frac{V_0^+}{Z_0} e^{j \cdot \beta \cdot l} - \frac{V_0^-}{Z_0} e^{-j \cdot \beta \cdot l}$$

$$Z_{in} = \frac{V(-l)}{I(-l)} \quad Z_{in} = Z_0 \cdot \frac{1 + \Gamma \cdot e^{-2j \cdot \beta \cdot l}}{1 - \Gamma \cdot e^{-2j \cdot \beta \cdot l}}$$

- the **input impedance** seen looking toward the load

$$Z_{in} = Z_0 \cdot \frac{(Z_L + Z_0) \cdot e^{j \cdot \beta \cdot l} + (Z_L - Z_0) \cdot e^{-j \cdot \beta \cdot l}}{(Z_L + Z_0) \cdot e^{j \cdot \beta \cdot l} - (Z_L - Z_0) \cdot e^{-j \cdot \beta \cdot l}}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

# The lossless line

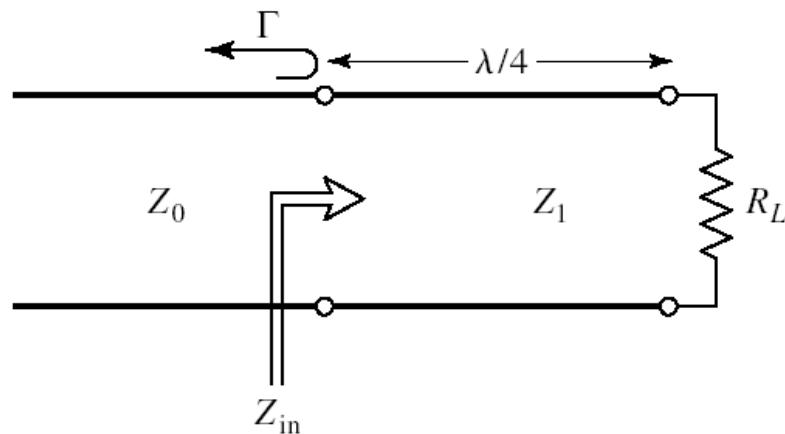
- the **input impedance** seen looking toward the load

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

# **Short theory for Lab 1**

# The quarter-wave transformer

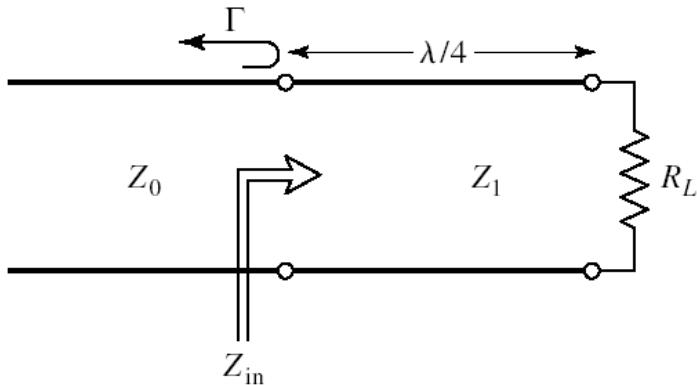
- Feed line – input line with characteristic impedance  $Z_0$
- (**Real!**) load impedance  $R_L$
- We desire matching the load to the feed line with a second line with the length  $\lambda/4$  and characteristic impedance  $Z_1$



$$\Gamma_{in} = 0 \Rightarrow Z_1 = \sqrt{Z_0 R_L}$$

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan(\beta l)}{Z_1 + jR_L \tan(\beta l)}$$

# The quarter-wave transformer



$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{in} = \frac{Z_1^2}{R_L}$$

$$\Gamma_{in} = \frac{Z_1^2 - Z_0 \cdot R_L}{Z_1^2 + Z_0 \cdot R_L}$$

$$\Gamma_{in} = 0 \Rightarrow Z_1 = \sqrt{Z_0 R_L}$$

- In the feed line ( $Z_0$ ) we have only progressive wave
- In the quarter-wave line ( $Z_1$ ) we have standing waves

# The lossless line

$$V(z) = V_0^+ \cdot (e^{-j\beta z} + \Gamma \cdot e^{j\beta z})$$

$$I(z) = \frac{V_0^+}{Z_0} \cdot (e^{-j\beta z} - \Gamma \cdot e^{j\beta z})$$

- time-average Power flow along the line

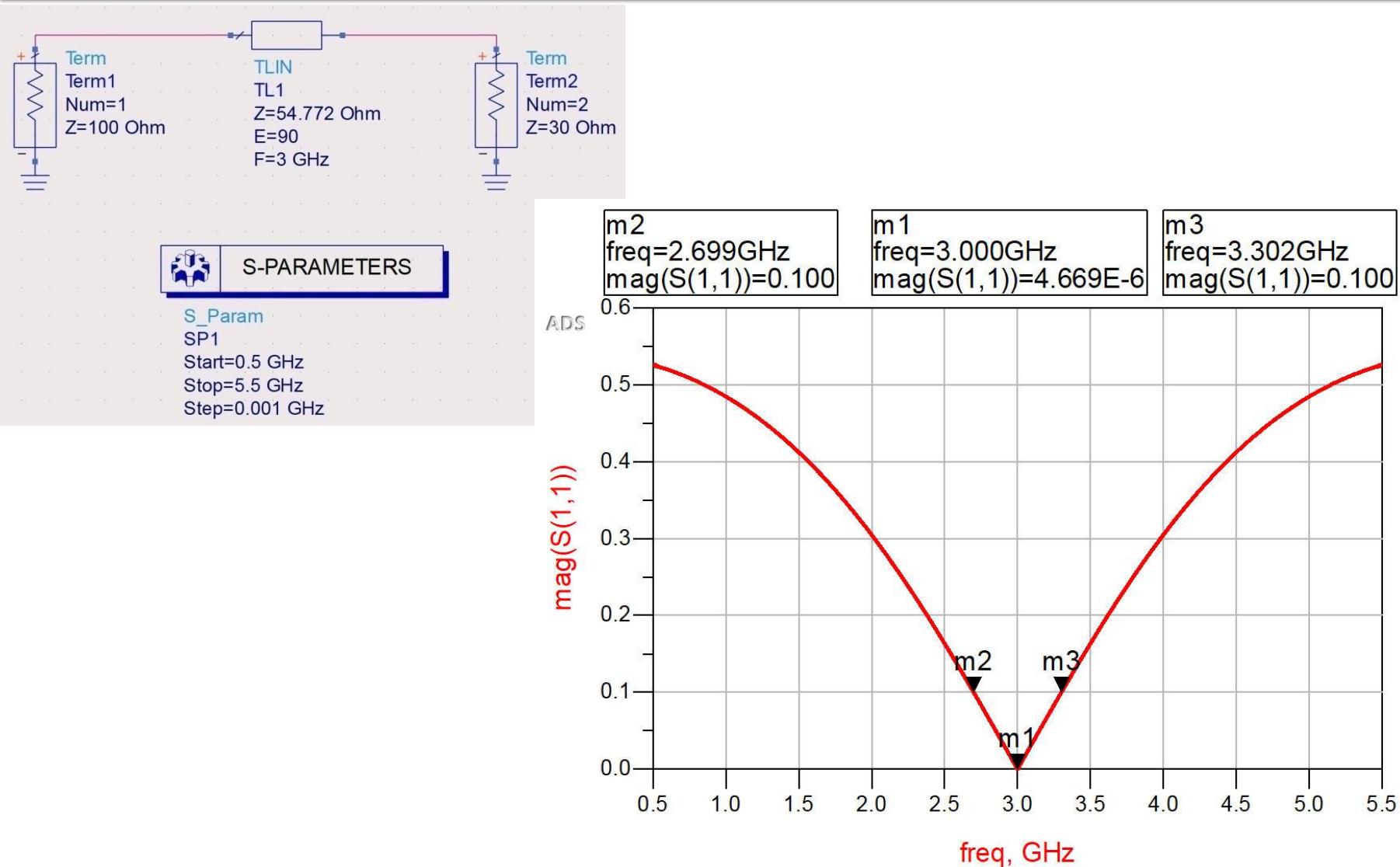
$$P_{avg} = \frac{1}{2} \cdot \text{Re}\{V(z) \cdot I(z)^*\} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot \text{Re}\left\{1 - \Gamma^* \cdot e^{-2j\beta z} + \Gamma \cdot e^{2j\beta z} - |\Gamma|^2\right\}$$

$$P_{avg} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot (1 - |\Gamma|^2)$$

$$(z - z^*) = \text{Im}$$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB] 
$$\text{RL} = -20 \cdot \log|\Gamma| \quad [\text{dB}]$$

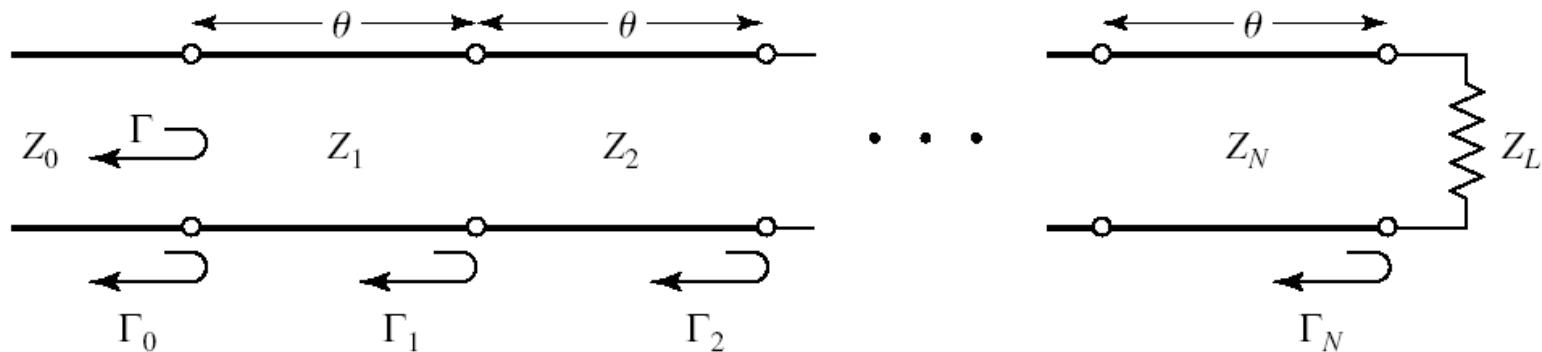
# The quarter-wave transformer



# Multisection Impedance Transformer

- The quarter-wave transformer can match any real load to any feed line impedance
- If a greater bandwidth for the match is required we must use multiple sections of transmission lines transformers:
  - binomial
  - Chebyshev

# Multisection Impedance Transformers



- We assume that all impedances **increase or decrease monotonically** across the transformer
- This implies that all reflection coefficients will be real and of the same sign
- Previously, 1 section  $\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta} \Rightarrow$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

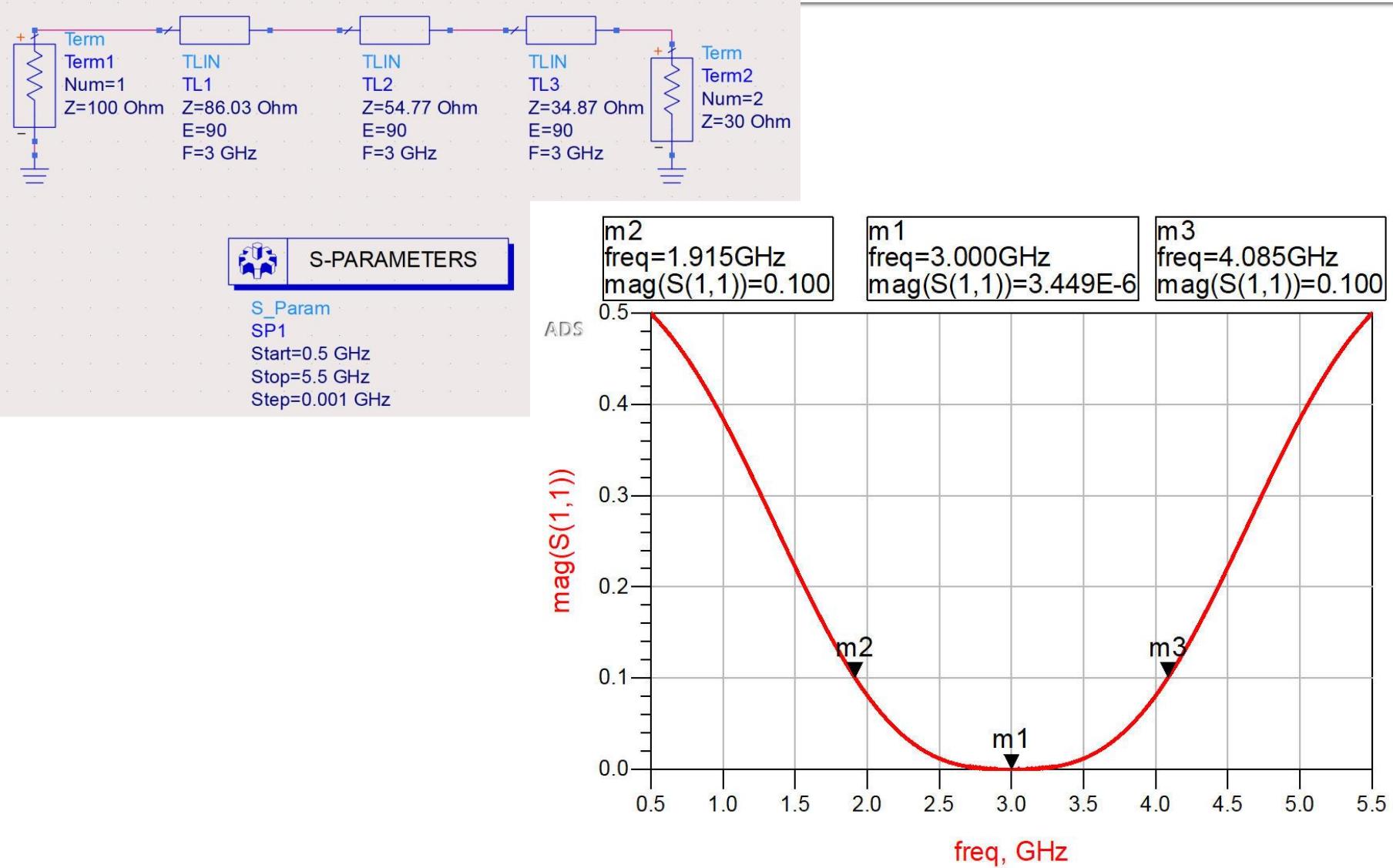
$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

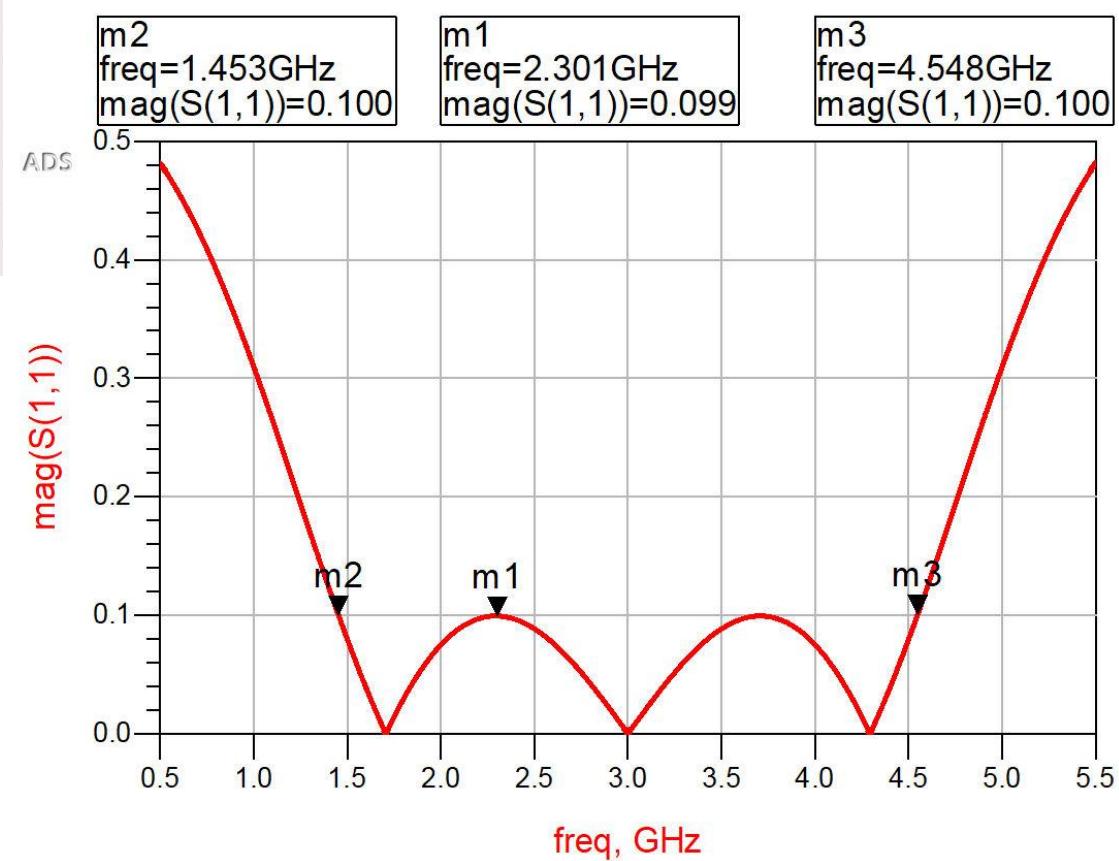
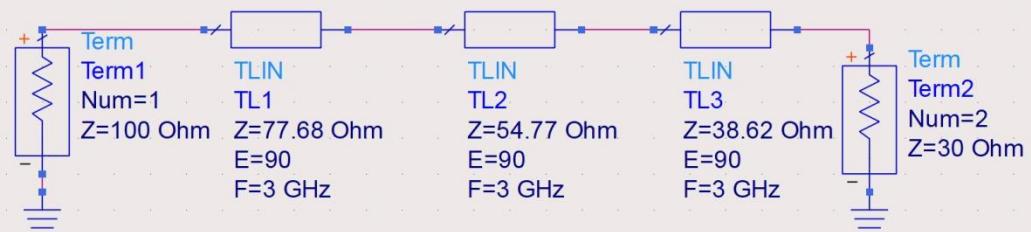
$$n = \overline{1, N-1}$$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

# Binomial



# Chebyshev



# Contact

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